14.1829.

A

NEW AND IMPROVED

SYSTEM OF CALCULATION,

IN WHICH

A UNIVERSAL RULE OF PROPORTION

IS, BY NEW ARRANGEMENT, APPLIED TO

QUESTIONS RELATING TO MILITARY AFFAIRS, MENSURATION, NATURAL PHILOSOPHY, AND MERCANTILE OPERATIONS.

BY DANIEL DOWLING,

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DEDICATED, BY PERMISSION, TO

HIS GRACE THE DUKE DE GUICHE.

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DEDICATION

TO

HIS GRACE THE DUKE DE GUICHE.

MY LORD DUKE,

In bringing the following work before the public under the sanction of your distinguished name, I am not influenced so much by your elevated rank, as the heir of hereditary honours, transmitted to you through a long line of illustrious warriors and statesmen, nor by the rare combination of public spirit, private virtues, and military glory, which render the Duke de Guiche conspicuous among the Peers of France, as by my lively recollection of the conciliating, amiable, and kind disposition, the dignified deportment, and the

ardent love of science, which distinguished the Count de Grammont from the rest of my Pupils.

In testimony of my high admiration of your transcendent talents, of my unfeigned esteem for your private worth, and of my gratitude for your many condescending favours, this work is respectfully inscribed to your Grace, by,

My Lord Duke,

Your Grace's most devoted

and humble Servant,

DANIEL DOWLING.

9, Upper Mall, Hammersmith;
January, 1829.

PREFACE.

The following sheets, containing an extended application of a general Rule of Proportion, published by the Author as an Appendix to the last edition of his Improved System of Arithmetic, were composed exclusively for the use of his Pupils. It was not, originally, his intention to publish them; but, finding by experience that they formed a system of arithmetical calculations more comprehensive, concise, and practically useful, than any hitherto offered to the public; and being informed that a gentleman, who had received some information on the subject from the Author's Pupils, had been lecturing on this system, *as his own*;* he considered it a duty he owed to himself and the public, to publish them, with appropriate explanations and illustrations.

* A Mr. Perry sent one of his *Travellers* to the Author, at the Mansion House, Highgate, to solicit the introducing of the "*Perrian System*," into his Establishment. The Author replied that he should previously ascertain whether the "*Perrian System*" was better than his own; but, if he could not produce six pupils, all under twelve years of age, who were superior to Mr. Perry in calculation, he would adopt his system. Mr. Perry accepted the challenge, and, after the pupils had solved questions which he could not even comprehend, he acknowledged, in their presence, *that he knew nothing of the system; that he had never seen it before; that he had no system to offer equal to it; and that he had never before seen any thing that approached it in excellence*. After that avowal, in the presence of his pupils, the Author permitted them to explain to him the principles upon which his system was founded, and made him a present of his "Improved System of Arithmetic," and the Appendix, in which the principles had been developed. Mr. Perry has, since that period, lectured on this system, *as his own*. Should he deny these *facts*, the Author pledges himself to bring forward the young gentlemen in whose presence he had made the above declaration.

Arithmetic is unquestionably one of the most useful branches of the Mathematics, and, when treated as a science, yields to none of them in clearness, precision, and in conducting to accurate and interesting results. Elementary works on this subject being considered by Mathematicians, *in this country*, as unworthy of their attention, have been committed to the care of men, who, being imperfectly acquainted with the first principles of the science, have crowded them with gross absurdities, unintelligible rules, and useless examples, that never could have been tolerated had not this branch of education been neglected by men of science.

The manner in which this branch of the Mathematics is generally taught is no less defective. On the Continent, men of the greatest eminence are practical teachers of Arithmetic; but, in this country, it is assigned to the writing-master, whose province is to examine the solutions, (which he generally does by the assistance of a Key,) and to see them set in a ciphering-book; but it forms no part of his duty to analyze the questions, and explain to the Pupils the principles upon which the operations are founded. Hence, in private schools, youth are uselessly employed the greatest part of the time they are at school in the acquisition of Arithmetic; and, in public grammar schools, it is entirely disregarded.

The object of the author has been to treat the subject as a science; to reduce it to a few simple principles; to substitute one general rule of proportion, easily understood and remembered, for the numerous rules that have perplexed and obscured the subject: how far he has succeeded, he leaves to others to determine.

If this system offered no other advantage than the removal of the distinction between the *Rule of Three direct, the Rule of Three inverse, the Rule of Three direct in Vulgar Fractions, the Rule of Three inverse*

in Vulgar Fractions, the Rule of Three direct in Decimals, the Rule of Three inverse in Decimals, and the Rule of Five or Compound Proportion, that would not be inconsiderable. But it is presumed that the arranging of Interest, Commission, Brokerage, Profit and Loss, Barter, Fellowship, the calculations of the Public Funds, Annuities, and Exchanges, under the general form, as applications of the Rule of Proportion, will be deemed an additional advantage.

The Author has not confined the application of his system to mercantile operations, he has extended it to the mensuration of solids and superficies, to mechanics, and other branches of Natural Philosophy.

In Fractions no distinction has been made between Compound Fractions and Multiplication, nor between Complex Fractions and Division; and questions involving Multiplication and Division of Fractions have been solved by one operation. Decimal Fractions should be taught with the first four operations of arithmetic, as the numerators only are used, the denominators being understood.

Many of the theorems and examples have been deduced from Hutton, Gregory, and Dalby's Mathematics, Kelly's Cambist and Merchants' Books; but the author has solved the questions by his own rule. He is indebted to Capt. Henry Kater for Theorem page 82. In the calculation in Mechanics, friction and other considerations have been disregarded.

THE AUTHOR RECOMMENDS THE FOLLOWING RULES TO TEACHERS.

I. Read the Question aloud to the Pupils, and point out to them the terms of the supposition and those of the demand.

II. Separate the supposition into first cause and first effect, and the demand into second cause and second effect.

III. Draw two lines at right angles to each other; place the first cause and first effect at each side of the vertical line, and the second cause and second effect in the vertical opposite angles, that is, the second cause vertically opposite the first cause, and the second effect vertically opposite the first effect, and put x instead of the quantity required.

IV. Do not permit the pupils to cancel until they can analyze the question, and arrange the terms in proportion.

V. Cancel the terms on one side of the vertical line with those on the other, that is, the terms of the first cause and second effect with those of the second cause and first effect; and divide the product of the terms that remain on the same side as x into the product of those that remain on the other side, for the result or value of x.

VI. Vary the question as often as it has conditions, by substituting the value for x, and x for another term; and make the Pupils write on their slates the questions that such alterations would produce.

Thus, Example 3, *page* 7,

Substituting 96 for x, and x for any other term as in the supposition, 4 feet long, it will be resolved into the following question.

If an iron bar, 6 feet long, 4 inches broad, and 2 inches thick, weigh 96 lb., how long must the bar be that is 3 inches broad, $1\frac{1}{2}$ inches thick, to weigh 36 lb. A great advantage will arise from this mode of investigation: every question being varied as often as it contains conditions, the Pupils will form different questions requiring different results. Frequently the variation produces questions more difficult than the original.

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AN

IMPROVED SYSTEM

OF

ARITHMETIC.

RATIOS AND PROPORTIONS.

RATIO is the relation which two magnitudes, of the same kind, have to one another, with respect to quantity; the first is called the *antecedent or cause*, and the second the *consequent or effect*. This relation is ascertained by examining how often the consequent is contained in the antecedent, and the quotient is termed *the exponent or value of the ratio*. Thus the value of the ratio between 12 and 4 is 3, and that between 4 and 12 is $\frac{1}{3}$; hence the value of the ratio between 4 and 12 is the reciprocal of that between 12 and 4.

The antecedent and consequent are called the *terms* of the ratio.

When the value of the ratio between two terms is equal to that between two other terms, they are said to be proportional; that is, the first is said to be to the second as the third is to the fourth: thus, $8 \div 2 = 12 \div 3$; then 8 are to 2 as 12 are to 3; and they are generally expressed by placing *two dots* between the first and second terms, *four* between the second and third, and *two* between the third and fourth: thus, $8 : 2 :: 12 : 3$; again, $\frac{1}{8} : \frac{1}{2} :: 3 : 12$, and $8 : 2 :: \frac{1}{3} : \frac{1}{12}$; that is $\frac{1}{8} \div \frac{1}{2} = 3 \div 12$, and $8 \div 2 = \frac{1}{3} \div \frac{1}{12}$.

THEOREM I.

If four numbers be proportional, the product of the extremes is equal to that of the means. Thus, $8 : 2 :: 12 : 3$; then $8 \times 3 = 2 \times 12$. (Prop. 16, Book VI. Euclid.)

The first and fourth terms are called the extremes, and the second and third the means.

Cor. If either of the extremes be required, it is found by dividing the product of the means by the given extreme.

THEOREM II.

If there be any number of magnitudes such, that the first is to the second as the third is to the fourth, the fifth to the sixth as the fourth is to the seventh, the eighth to the ninth as the seventh is to the tenth, &c. putting the last term found for the third term in the succeeding proportion, then will the product of all the first antecedents be to the product of all the first consequents, as the second antecedent is to the last consequent found. Thus—

$$\begin{array}{l} 2 : 4 :: 6 : 12 \\ 3 : 9 :: 12 : 36 \\ \frac{1}{2} : \frac{1}{4} :: 36 : 18 \end{array}$$

Then will $2 \times 3 \times \frac{1}{2} : 4 \times 9 \times \frac{1}{4} :: 6 : 18.$

$$\therefore \frac{6}{2} : \frac{36}{4} :: 6 : 18.$$

Or, $3 : 9 :: 6 : 18.$

Every question contains two parts, viz. the *hypothesis* or *supposition*, and the *demand*; the former begins with one of the words, *if at*, &c.; and the latter with the words, *how much*, *how many*, *at what rate*, &c.

The supposition and demand are again divided into causes and effects, or into the first cause and the first effect, the second cause and the second effect.

Causes are of the same kind, and effects are of the same kind.

Whatever influences the cause must be considered as forming a part of it. Also, whatever influences the effect must be considered as belonging to the effect.

In every question there are always given the first cause and the first effect, and the second cause *or* second effect, or a part of either, is always required.

EXAMPLES FOR ILLUSTRATION.

Example 1.

If 3 yards of cloth cost 15 shillings, how much will 6 yards cost?

Here 3 yards and 15 shillings form the supposition, 6 yards and the required shillings the demand, again 3 yards of cloth are the first cause, 15 shillings the first effect; 6 yards of cloth the second cause, and the number of shillings required the second effect.

Example 2.

If 3 men in 6 days dig a ditch 30 yards long and 3 feet broad, how many men would be required to dig a ditch 60 yards long and 4 feet broad in 3 days?

Here 3 men and 6 days form the first cause, 30 yards long and 3 feet broad of ditch, form the first effect; 60 yards long and 4 feet broad of ditch, form the second effect; and 3 days, together with the number of men required, form the second cause.

Example 3.

How many yards of cloth $\frac{7}{4}$ of a yard wide will line 14 yards of broad cloth $\frac{5}{4}$ of a yard wide?

Here 14 yards of broad cloth, and $\frac{5}{4}$ of a yard, are the first cause; the first effect, is the superficial contents of the cloth. The second cause producing the same effect, is a certain number of yards (required) and $\frac{7}{4}$ of a yard wide; and the second effect, is the same as the first.

Causes of different kinds sometimes produce the same effect, and then a part of the second cause is required.

Example 1.

How many yards of cloth, at $18\frac{1}{2}$d. per yard, can be bought for $308\frac{1}{3}$ lb. of pepper, at 15d. per lb.?

Here $308\frac{1}{3}$lb. of pepper, at 15d.per lb. are to produce the same effect as a certain number of yards (required) at $18\frac{1}{2}$d. per yard; therefore $308\frac{1}{3}$ lb. and 15d. are the first cause, producing the first effect, $18\frac{1}{2}$d., and a certain number of yards of cloth (to be found) constitute the second cause, producing a second effect equal to the first.

Example 2.

How many pounds of tea, at 8s. a pound, are equivalent to 23 lb. at 6s. per lb.?

Here the effect produced by the 23 lb. at 6s. per lb. is equivalent to the effect produced by a certain number of pounds of tea (to be found) at 8s. per lb.; consequently, 23 lb. and 6s. are the first cause, their amount, the first effect; the number of pounds of tea required, and 8s. the second cause, their amount the second effect, which is equal to the first.

EFFECTS VARY AS THEIR CAUSES.

Example 1.

If 3 yards of cloth cost 15s., 6 yards will cost 30s.; that is, as much as 6 yards exceed 3 yards, so much will the price of 6 yards exceed the price of 3 yards; therefore, by proportion as 3 yds. : 6 yds. :: 15s. : 30s. and (by Theorem 1,) $3 \times 30 = 6 \times 15$, that is, the product of the first cause and second effect, is equal to the product of the first effect and second cause. Hence we deduce the following

RULE.

Draw two right lines, cutting each other, as A B and C D, and forming the angles *a*, *b*, *c*, *d*. In angle *a* put the first cause, in angle *b* the first effect, in angle *c* the second cause, and in angle *d* the second effect, and put x for the quantity required.

	A	
First cause, 3 yards. *a*		*b* 15s. first effect.
C———		———D
Second effect, x s. *d*		*c* 6 yds. second cause.
	B	

As the products of the quantities on each side of the line A B are equal, to cancel the numbers common to both sides, does not affect the equality, for magnitudes have the same ratio to one another which their equimultiples have, (Prop. 15, book V. Euc.), we therefore divide any two of them by their common divisor, and use the quotients in their stead; we continue the operation until there are no numbers on each side of A B that have a common divisor, then

divide the product of all the numbers on the same side as x into the product of all the numbers on the other side, for the answer.

Example 2.

If 16 men in 3 days dig a ditch 60 yards long and 4 feet broad, how many yards of a ditch 3 feet wide would 3 men dig in 6 days?

Arrangement.

$$
\begin{array}{lr|ll}
 & \multicolumn{2}{c}{\text{A}} & \\
\text{First cause,} & \left\{\begin{array}{r}\text{men, } 16\\ \text{days, } 3\end{array}\right. & \left.\begin{array}{l}60\text{ yds.}\\ \;\;4\text{ ft.}\end{array}\right\} & \text{first effect.}\\
 & \text{C}\text{———} & \text{———D} & \\
\text{Second effect,} & \left\{\begin{array}{r}\text{yds. } x\\ \text{feet, } 3\end{array}\right. & \left.\begin{array}{l}3\text{ men}\\ 6\text{ days}\end{array}\right\} & \text{second cause.}\\
 & \multicolumn{2}{c}{\text{B}} &
\end{array}
$$

Operation.

$$
\begin{array}{r|l}
4\;\;1 & 15\\
\text{C}\left\{\begin{array}{r}\text{men, } \cancel{16}\\ \text{days, } \cancel{3}\\ 1\end{array}\right. & \left.\begin{array}{l}\cancel{60}\text{ yds.}\\ 1\\ \cancel{4}\text{ feet.}\end{array}\right\}\text{E}\\
\hline
\text{E}\left\{\begin{array}{r}\text{yds. } x\\ \\ \text{feet, } \cancel{3}\end{array}\right. & \left.\begin{array}{l}\cancel{3}\text{ men}\\ 1\\ \cancel{6}\text{ days}\end{array}\right\}\text{C}\\
1 & 2
\end{array}
$$

$$\therefore x = 15 \times 2 = 30 \text{ yds. Ans.}$$

EXPLANATION.

We cancel 16 and 60 by dividing each by 4, and retain the quotients 4 and 15; we again cancel the 4 obtained from the 16, and the 4 feet, by dividing each by 4, the quotients are 1 and 1; we next cancel

3 days and 6 days, by dividing each by 3, and obtain 1 and 2; and, lastly, cancel 3 feet and 3 men, which gives 1 and 1. Now, as only 1 remains on the same side as x, and 15 and 2 on the other side, $x = 15 \times 2 = 30$ yards, the answer.

Example 3.

If an iron bar 4 feet long, 3 inches broad, and $1\frac{1}{2}$ inches thick, weigh 36 lb. how much will a bar weigh that is 6 feet long, 4 inches broad, and 2 inches thick?

Arrangement.

$$\begin{array}{ll|l} \text{C} & \left\{\begin{array}{l}\text{ft. } 4\\ \text{in. } 3\\ \text{in. } \frac{3}{2}\end{array}\right. & 36\text{ lb. E} \\ \hline \text{E} & \qquad x & \left.\begin{array}{l} 6\text{ ft.}\\ 4\text{ in.}\\ 2\text{ in.}\end{array}\right\}\text{C}\end{array}$$

Operation.

$$\begin{array}{r|l} \text{ft. } 1\ \cancel{4} & \cancel{9}\ \cancel{3}\ 1 \\ \text{in. } 1\ \cancel{3} & \cancel{36}\text{ lb.} \\ \text{in. } 1\ \cancel{3} & 2 \text{ denom. of } \frac{3}{2} \\ \hline & 6 \\ x & 4 \\ & 2 \end{array}$$

$$x = 2 \times 6 \times 4 \times 2 = 96 \text{ lb. Ans.}$$

EXPLANATION.

We cancel 4 feet and 36 lb. by dividing by 4, and obtain 1 and 9; again, divide 3 inches and 9 by 3, and obtain 1 and 3; and, lastly, divide 3 inches, and the 3 obtained from the last division, by 3, and as all the numbers on the same side as x are cancelled, $x = 2 \times 6 \times 4 \times 2 = 96$, answer.

REMARK.—It is evident that in removing the denominator of a fraction to the other side of the perpendicular line, we multiply both sides by that quantity, and therefore do no affect the equality.

MULTIPLICATION AND DIVISION OF FRACTIONS.

Unity is to the multiplier, as the multiplicand is to the product. (Cor. 1, page 198, part 1.)

Example 1.

Multiply $\frac{1}{2}$ of $\frac{15}{2}$ of $\frac{3}{4}$ by $\frac{7}{8}$ of $\frac{1}{2}$ of $\frac{8}{9}$ of $\frac{9}{10}$.

Arrangement.

1	$\frac{1}{2}$
	$\frac{15}{2}$
	$\frac{3}{4}$
x	$\frac{7}{8}$
	$\frac{1}{2}$
	$\frac{8}{9}$
	$\frac{9}{10}$

Operation.

1	1	
2	~~15~~	3
2	3	
4		
x		
~~8~~	7	
2	1	
~~9~~	~~8~~	
~~10~~		
2		

$$64 \times x = 7 \times 3 \times 3 = 63.$$
$$x = \tfrac{63}{64}. \text{ Ans.}$$

The divisor is to unity, as the dividend is to the quotient. (Cor. 2, page 198, part 1.)

Example 2.

Divide $\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{1}{5}$ by $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$.

Arrangement.

$$
\begin{array}{c|c}
\frac{3}{5} & \\
\frac{5}{6} & 1 \\
\frac{7}{8} & \\
\hline
 & \frac{7}{8} \\
x & \frac{4}{5} \\
 & \frac{3}{7} \\
 & \frac{1}{8}
\end{array}
$$

Operation.

$$
\begin{array}{c|c}
\cancel{3} & \cancel{5} \\
\cancel{5} & 6 \\
\cancel{7} & \cancel{8} \\
\hline
\cancel{8} & \cancel{7} \\
5 & 4 \\
7 & \cancel{3} \\
5 & 1 \\
x &
\end{array}
$$

$$5 \times 7 \times 5 \times x = 6 \times 4$$
$$175 \times x = 24$$
$$\therefore x = \tfrac{24}{175}. \text{ Ans.}$$

In the last example we divide the product of $\frac{7}{8}$, $\frac{4}{5}$, $\frac{3}{7}$, and $\frac{1}{5}$, by the product of $\frac{3}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.

Ob. 1.—It is evident that Multiplication and Division of Fractions can be performed by one operation.

Ob. 2.—The distinction between a compound fraction,—that is, the fraction of a fraction,—and multiplication, is absurd; they are identically the same. The same observation may be applied to complex fractions and division, that is, fractional numerators and denominators.

Thus $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{6}$ of $\frac{8}{9} = \frac{1}{2} \times \frac{3}{4} \times \frac{7}{8} \times \frac{8}{9}$

and $\dfrac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \div \frac{7}{8}$

Example 3.

A regiment of soldiers, consisting of 976 men, are to be new clothed, each coat containing $2\frac{1}{2}$ yards of scarlet cloth of $1\frac{5}{8}$ yard wide, is to be lined with shalloon of $\frac{7}{8}$ yard wide, how many yards of the shalloon will line the whole?

Arrangement		*Operation.*	
Long $2\frac{1}{2}$ Broad $1\frac{5}{8}$	1 Coat	5 $\cancel{2}$ 13 $\cancel{8}$	2 8
Coats, 976	x long $\frac{7}{8}$ broad	$\cancel{8}$ $\cancel{976}$ 488	7 8 x

Then $488 \times 13 \times 5 \div 7 = x = 4531\frac{3}{7}$ yds. Ans.

EXAMPLES FOR PRACTICE ON MULTIPLICATION AND DIVISION OF FRACTIONS, TO BE DONE IN ONE OPERATION.

1. Multiply $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{6}{7}$, $\frac{4}{5}$, together, and divide the product by $\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{4}{5}$. Ans. $\frac{9}{5} = 1\frac{4}{5}$.

2. Multiply $3\frac{1}{2}$, $\frac{4}{5}$, $5\frac{1}{6}$, $\frac{6}{7}$, together, and divide the product by $\frac{4}{5}$ of $\frac{3}{4}$ of $5\frac{1}{2}$. Ans. $\frac{124}{33} = 3\frac{25}{33}$.

3. Reduce $\dfrac{\frac{1}{2} \text{ of } \frac{3}{4} \text{ of } \frac{5}{6} \text{ of } \frac{6}{7} \text{ of } \frac{1}{3}}{\frac{1}{2} \text{ of } \frac{4}{5} \text{ of } \frac{5}{6} \text{ of } \frac{7}{8}}$ to a simple fraction. Ans. $\frac{15}{49}$.

4. Reduce $\dfrac{5\frac{1}{2} \text{ of } 6\frac{2}{3} \text{ of } 4\frac{1}{2}}{3\frac{3}{4} \text{ of } 6\frac{2}{3} \text{ of } 8\frac{3}{4}}$ to a simple fraction. Ans. $\frac{132}{175}$.

5. If $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a yard of cloth cost $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{1}{2}$ of a pound, how much will the $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of 50 yards cost? Ans. £56 19 $3\frac{7}{8}$.

6. If the $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of 50 yards of cloth, cost the $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{7}$ of $\frac{49}{3}$ of £150, how much will the $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{6}{7}$ of 7000 yards cost? Ans. £48000.

PROMISCUOUS QUESTIONS.

Example 1.

A ditch 40 yards long, 13 feet broad, and 5 feet deep, was dug in 6 days by 8 men, working 7 hours a day, how many hours a day for 13 days must 9 men work, to dig a ditch 75 yards long, 14 feet wide, and 4 feet deep?

Arrangement and Operation.

	5
Men, ~~8~~	~~40~~ yards long.
Days, ~~6~~ 2	13 feet broad.
Hours, 7	~~5~~ feet deep.
3 ~~15~~	x hours.
Yards long, ~~75~~	13 days.
Feet wide, 14	~~9~~ men.
Feet deep, 4	~~8~~

$$4\times14\times7\times2=13\times13\times x$$

$$\tfrac{784}{169}=x=4\tfrac{108}{169}\text{ hours. Ans.}$$

Example 2.

If 4 compositors, in 16 days, of 12 hours long, can compose 14 sheets, of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line,—in how many days, of 10 hours long, can 9 compositors compose a book, to be printed with the same letter, consisting of 30 sheets, of 16 pages in a sheet, 48 lines in a page, and 45 letters in a line?

Arrangement and Operation.

Compositors,	~~4~~	~~2~~	~~14~~	7 sheets.
Days,	~~16~~	~~8~~	~~24~~	~~2~~ pages.
		~~4~~		11
Hours,	~~12~~		~~44~~	lines.
			~~40~~	~~4~~ letters.
				~~2~~
Sheets,	~~30~~	~~3~~		x days.
Pages,	~~16~~	~~8~~	~~10~~	~~2~~ hours.
Lines,	48		~~9~~	compositors.
Letters,	~~45~~		~~3~~	
	~~15~~			
	3			

$$8 \times 3 \times 48 = 7 \times 11 \times x \therefore \frac{1152}{77} = 4\frac{74}{77} \text{ days: Ans.}$$

Example 3.

If a quantity of provisions serve 1400 men 20 weeks, at the rate of 14 ounces each man per day, how many men will the same provisions maintain for 32 weeks at the rate of 8 ounces each man per day?

Arrangement and Operation.

	175			
Men, -	~~1400~~			
Weeks,	~~20~~ 5	Provisions.		
Oz. -	~~14~~			
	7			
			x	men.
				4
Provisions			~~32~~	~~8~~ weeks.
			~~8~~	ounces.

$$7 \times 5 \times 175 = 4 \times x$$

$$\frac{6125}{4} = 1531\tfrac{1}{4} = x. \text{ Ans.}$$

Here the first and second causes produce the same effect.

Example 4.

If a family of 13 persons spend £64 in butcher's meat in 8 months, when the meat is 6½d. per lb., how much would a family of 12 persons spend in 9 months, when the meat is 7d. per lb.?

Arrangement and Operation.

$$\begin{array}{r|l}
\text{Persons, - } 13 & \\
\text{Months, - } \cancel{8} & 8 \\
\text{Per lb. } 6\tfrac{1}{2} = \dfrac{13}{\cancel{2}} & \pounds\cancel{64} \\
 & 2 \\
\hline
 & \text{12 persons.} \\
x & \text{9 months.} \\
 & \text{7d. per lb.}
\end{array}$$

$$13 \times 13 \times x = 8 \times 2 \times 12 \times 9 \times 7$$

$$169x = 12096$$

$\therefore$ $12096 \div 169 = £71$ 11s. 5d. $3\frac{1}{169}$ qrs. $= x$. Ans.

Example 5.

If I buy cloth at 15s. per yard, at how much must I sell it per yard to gain 25 per cent.?

Arrangement and Operation.

$$\begin{array}{c|c}
4 & 5 \\
\cancel{100} & \cancel{125} \\
\hline
x & \text{15s.}
\end{array}$$

$$x = \frac{15 \times 5}{4} = \frac{75}{4} = \text{18s. 9d. Ans.}$$

Example 6.

If a ton of wine be purchased for £50, ready money, and sold for £54 10, payable in 8 months, how much is gained per cent. per annum?

Arrangement and Operation.

$$
\begin{array}{r|l}
\pounds\cancel{50}. & 4\tfrac{1}{2} = \frac{9}{\cancel{2}} \\
2 & \\
\text{Months, } \cancel{8} & \\
\cancel{2} & \\
\hline
 & \cancel{2} \\
x & \pounds\cancel{100} \\
 & \quad\cancel{12}\text{ months.} \\
 & \quad 3
\end{array}
$$

$$2x = 27$$
$$x = \tfrac{27}{2} = 13\tfrac{1}{2}.\ \text{Ans.}$$

Example 7.

If goods be bought for £3 15, ready money, and sold for £4 6 3, payable in 18 months, how much is gained per cent. per annum?

£4	6	3
3	15	0
0	11	3 gained by £3 15, in 18 months.

Arrangement and Operation.

$$
\begin{array}{r|l}
 & \qquad \cancel{9} \\
3\tfrac{3}{4} = \frac{\cancel{15}}{4} & 11\tfrac{1}{4} = \frac{45}{4} \\
3\ \cancel{18} & \\
\hline
x & \cancel{100}\ 5 \\
\cancel{20} & \cancel{12}\ 2
\end{array}
$$

$$x = 2 \times 5 = 10 \text{ per cent.}$$

In this example the first effect is shillings, the first and second causes are pounds, the second effect (x) will be shillings; we therefore place 20 as a co-efficient or multiplier for x, and the answer is in pounds. If the given effect were pence, we should place 12 and 20, to have the answer in pounds. If the given cause were farthings, 4, 12, 20, are to be placed as multipliers for x, to have the answer in pounds.

Or thus,

$$\begin{array}{r|l} 5 & 3 \\ \text{Shillings, } \cancel{75} & \frac{\cancel{45}}{\cancel{4}}\text{ shillings.} \\ \text{Months, } \cancel{18} & \\ 6\ \cancel{2} & \\ \hline & 10 \\ x & \cancel{20} \\ & \cancel{100}\text{ shillings.} \\ & \cancel{12}\text{ months.} \\ & 3 \end{array}$$

$x = 10$. Ans.

Example 8.

If cloth be bought at 15s. 6d. per yard, and sold for 18s. how much is the gain per cent.?

18s.	0d.
15	6
2	6 gained by 15s. 6d.

Arrangement and Operation.

$$\begin{array}{r|l} 15\frac{1}{2}=\frac{31}{2} & \frac{5}{2} \\ \hline x & 100 \end{array}$$

$31 \times x = 500$

$x = \frac{500}{31} = 16\frac{4}{31}$. Ans.

Example 9.

What quantity of drugget, at 18½d. per yard, will be equivalent to 925 lb. of black tea, at 6s. per lb.?

Arrangement and Operation.

$$\begin{array}{r|l} 25 & \\ \cancel{925} & \\ 6 & \\ 12 & \\ \hline & x \\ & \cancel{37} \\ 2 & \cancel{2} \end{array}$$

$25 \times 6 \times 12 \times 2 = 3600 = x$. Ans.

Here the effect produced by the tea, and the price of it per lb. must be equivalent to the effect produced by the quantity of drugget required, at the rate of 18½d. per yard.

Example 10.

A. has coffee, which in barter with B. he charges at 10d. per lb. more than it cost him, for tea of which the prime cost was 10s. per lb., but the barter price 12s. 6d. At what rate had A. bought his coffee?

12s.	6d.
10	0
2	6 more than it cost B.

Arrangement and Operation.

$$\begin{array}{r|l} & 2 \\ 2\frac{1}{2} = \frac{\cancel{5}}{\cancel{2}} & \cancel{10}\text{s.} \\ & 2 \\ \hline x & 10\text{d.} \end{array}$$

$x = 40$d. $= 3$s. 4d. Ans.

Example 11.

A. has cloth which cost 8d. per yard, and charges in barter 13d., giving 9 months' credit ; B. has goods which cost him 12d. the lb., and gives him 6 months' credit in barter : how much must he charge his goods per lb. to make the barter equal?

$$\begin{array}{r|l} \text{Months, } \cancel{9} & 5\text{d.} \\ 3 & \\ \hline x & \cancel{6}\text{ months.} \\ & 2 \end{array}$$

$x=3\frac{1}{3}$, what A. ought to gain by 8d. at 6 months' credit; and 8d. $+ 3\frac{1}{3}$d. $=11\frac{1}{3}$d. what A. should charge, giving 6 months' credit.

Again,

$$\begin{array}{r|l} 2 & 11\frac{1}{3}=\frac{\cancel{34}}{\cancel{3}}\,.\,17 \\ \cancel{8}\text{d.} & \\ \cancel{3} & \\ \hline x & \cancel{12}\text{d.} \\ & 3 \end{array}$$

$x=17$d. Ans.

Or thus,

$$\begin{array}{r|l} 2 & \\ \cancel{8}\text{d.} & \\ \text{Months, } \cancel{9} & 5\text{d. gained.} \\ \cancel{3} & \\ \hline x & \cancel{12}\text{d.} \\ & \cancel{3} \\ & \cancel{6}\text{ months.} \\ & \cancel{2} \end{array}$$

$x = 5$d.

That is, B. should gain 5d. by 12d. in 6 months. Hence, $12+5=17$d. what B. should charge per lb. for his goods.

Example 12.

If A. charge 13d. per lb. (giving 9 months' credit) for goods of which the prime cost was 8d. per lb., at what rate per yard did B. purchase shirting, for which in barter he charges 17d. giving 6 months' credit?

This example is the converse of the last, in which we have shewn that A. should charge $11\frac{1}{3}$d. per lb. for his goods at 6 months' credit.

$$\begin{array}{c|c} \cancel{2}\ \dfrac{\cancel{34}}{\cancel{3}} & \cancel{8}\ 4 \\ & 3 \\ \hline & \\ x & \cancel{17} \end{array}$$

$x = 4 \times 3 = 12$. Ans.

Example 13.

Two gentlemen were partners in the purchase and sale of an estate, by which they cleared a profit of £276. The shares of the purchase money were in the ratio of $\frac{2}{3}$ to $\frac{1}{7}$; what is each partner's share of the profit?

$\frac{2}{3} = \frac{14}{21}$, $\frac{1}{7} = \frac{3}{21}$, $\therefore \frac{14}{21} + \frac{3}{21} = \frac{17}{21}$.

Then,

Arrangement.

$$\begin{array}{c|c} \dfrac{17}{21} & 276 \\ \hline & \\ x & \frac{1}{3} \end{array}$$

Operation.

$$\begin{array}{c|c} 17 & \cancel{21}\ 7 \\ & 276 \\ \hline \cancel{3} & 1 \\ x & \end{array}$$

$$x = \frac{276 \times 7}{17} = 227\tfrac{5}{17}, \text{ A.'s share.}$$

$276 - 227\frac{5}{17} = 48\frac{12}{17}$ B.'s

A.'s share £$227\frac{5}{17}$, and B.'s £$48\frac{12}{17}$. Ans.

Example 14.

M., N., O., and P. bought a ship for £4080, of which sum M. contributed £1500, N. 1080, O. 1000, and P. £500. The ship received damage on her first voyage, and was sold for £3100. It is required to assign to each individual his share of this money.

M.	1500
O.	1000
N.	1080
P.	500
	£4080 sum.

Arrangement and Operation.

$$\begin{array}{r|l} & 775 \\ \cancel{4080} & \cancel{3100} \\ \cancel{102} & \\ 17 & \\ \hline x & \cancel{1500} \\ & 25 \end{array}$$

$\therefore 25 \times 775 \div 17 = £1139 \quad 14 \quad 1\frac{7}{17}$, M.'s.

Again,

$$\begin{array}{r|l} \cancel{4080} & \cancel{1000} \\ 108 & 25 \\ \hline x & 3100 \end{array}$$

$x = \dfrac{25 \times 3100}{102} = £759 \quad 16 \quad 0\frac{16}{17}$, O.'s.

In like manner, N. and P.'s shares may be determined.

Example 15.

What is the interest of £547 15, for 3 years, at 5 per cent. per annum?

$$\begin{array}{r|l} 4 & \\ \cancel{20} & \\ \text{Principal } \cancel{100} & \cancel{5} \text{ interest.} \\ \text{Year - } 1 & \\ \hline & 109{\cdot}55 \\ x & \cancel{547{\cdot}75} \text{ principal.} \\ & 3 \text{ years.} \end{array}$$

$$x = \frac{109{\cdot}55 \times 3}{4} = £82{\cdot}1625 = £82 \;\; 3 \;\; 3. \text{ Ans.}$$

Example 16.

How much is the interest of £120 10, for 125 days, at 4 per cent. per annum?

$$\begin{array}{r|l} \cancel{100} & \\ 73 & \\ \cancel{365} & 4 \\ \hline x & \cancel{120{\cdot}5} \\ & 1{\cdot}205 \\ & \cancel{125} \\ & 25 \end{array}$$

$$x = \frac{25 \times 1{\cdot}205 \times 4}{73} = £1 \;\; 13 \;\; 0\tfrac{12}{73}. \text{ Ans.}$$

Example 17.

Calculate the interest of £547 15, for 10 years, at $2\frac{1}{2}$ per cent.

$$\begin{array}{r|l} 2 & \\ \cancel{20} & \frac{5}{\cancel{2}} \\ \cancel{100} & \\ 2 & \\ \hline 4 & 547\frac{3}{4}=\frac{2191}{4} \\ x & \\ & \cancel{10} \end{array}$$

$$x=\frac{2191}{16}=£136 \quad 18 \quad 6. \text{ Ans.}$$

Example 18.

In what time will £700 amount to £784, at 4 per cent. per annum?

£784—700=84 interest.

$$\begin{array}{lr|l} \text{Principal,} & \cancel{100} & \\ \text{Year,} & 1 & \cancel{4} \text{ interest.} \\ \hline & & x \text{ years.} \\ \text{Interest,} & \cancel{84} & \cancel{700} \text{ principal.} \\ & \cancel{12} & \cancel{7} \\ & 3 & \text{Years, } 3=x. \text{ Ans.} \end{array}$$

Example 19.

Required the principal to gain £200 in 7 years, at 3 per cent. per annum?

Arrangement and Operation.

$$\begin{array}{lr|l} \text{Principal,} & 100 & \\ \text{Year,} & 1 & 3 \text{ interest.} \\ \hline & & x \text{ principal.} \\ \text{Interest,} & 200 & 7 \text{ years.} \end{array}$$

$$200\times 100=21\times x=\frac{20000}{21}=952\frac{8}{21}=x. \quad \text{Ans.}$$

6

Example 20.

At what rate per cent. will £198 gain £22 10, in 2 years?

Arrangement and Operation.

22	5
Principal, ~~198~~	$\frac{\cancel{45}}{\cancel{2}}=22\frac{1}{2}$ interest.
Years, ~~2~~	
~~2~~	
x	~~100~~ principal.
	~~50~~ 25
	1 year.

$$22\times x=5\times 25$$
$$x=\tfrac{125}{22}=5\tfrac{15}{22}\text{ per cent. Ans.}$$

Example 21.

If an agent dispose of goods to the amount of £4863 12, what is his commission at $2\frac{1}{2}$ per cent.?

Arrangement and Operation.

~~100~~	2·5
x	~~4863·6~~
	48·636

$$x=48{\cdot}636\times 2{\cdot}5=£121{\cdot}59.\text{ Ans.}$$

Here we cancel 100, by removing the decimal point two places towards the left hand.

Example 22.

How much is the brokerage, at 4s. 1d. per cent. on the sale of 18 pipes of wine, at £93 10, per pipe?

~~£100~~	49d.
	3
	~~18~~ pipes.
x	~~93·5~~
~~240~~d.	~~·935~~
~~40~~	·023375

$x = \cdot023375 \times 3 \times 49 = £3\cdot436125 = £3 \quad 8 \quad 7$. Ans.

Example 23.

How much is the commission on £7345 10, at $2\frac{1}{2}$ per cent.?

~~100~~	~~2·5~~
~~40~~	
8	
x	~~7345·5~~
	1469·1

$x = \frac{1469\cdot1}{8} = £183\cdot6375$. Ans.

Example 24.

How much stock can I purchase in the 3 per cent. reduced annuities, at $62\frac{1}{2}$ per cent. for £700?

6

$$
\begin{array}{r|l}
\cancel{62\cdot5} & \cancel{100} \\
\cancel{\cdot625} & \\
\cancel{\cdot125} & \\
\hline
\cdot025 & \pounds\cancel{700} \\
x & \cancel{140} \\
 & 28
\end{array}
$$

$x = \frac{28}{\cdot025} = £1120$. Ans.

See note, page 240, part 1.

Example 25.

Calculate the dividend on £1976 15, in the 3 per cent. reduced.

$$
\begin{array}{r|l}
\text{Months, } \cancel{2} & \\
\cancel{12} & \pounds3 \\
\pounds\cancel{100} & \\
\hline
 & \cancel{1976\cdot75} \\
x & \cancel{19\cdot7675}9\cdot88375 \\
 & \cancel{6}
\end{array}
$$

$x = 9\cdot88375 \times 3 = £29\cdot65125 = £29 \quad 13$. Ans.

Example 26.

How much will be received for £5000 stock in the 3 per cent. consols, at $74\frac{1}{2}$ per cent.?

$$
\begin{array}{r|l}
\cancel{100} & 74\cdot25 \\
\hline
x & 50\cancel{00}
\end{array}
$$

$x = 74\cdot25 \times 50 = 742\cdot5 \times 5 = £3712\cdot5 =$
£3712 10. Ans.

See note, page 240, part 1.

Example 27.

How much will £9764 Navy 5 per cent. Annuities, come to, at 95¼ per cent.?

$$\begin{array}{r|l} \cancel{100} & \cdot 95 \\ \hline x & 9764 \\ & \quad\cdot 95 \end{array}$$

$$\begin{array}{r} 48820 \\ 87876 \\ \hline £9275\cdot80 = £9275\ 16.\ \text{Ans.} \end{array}$$

Example 28.

Reduce 1000 guilders to sterling money, exchange at 10 florins 10 stivers per pound sterling.

$$\begin{array}{r|l} \text{Florins, } \cancel{10.5} & £1 \\ \cdot 105 & \\ \hline £x & 10\cancel{00} \text{ florins.} \end{array}$$

$x = \cdot\frac{10}{105} = £95\cdot238$. Ans.

Example 29.

Required the value, in Flemish money, of £350 sterling, exchange at 35s. 6d. Flemish.

Sterling, £1	~~426d.~~ Flemish. 71
Flemish, £x	
Flemish, ~~240d.~~ 4	£35~~0~~ sterling.

$$x=\frac{71\times35}{4}=£621\tfrac{1}{4}\text{ Flemish. Ans.}$$

Example 30.

The course of exchange between London and Madrid is 33d. per piastre, between London and Oporto 65d. per milree, and between Oporto and Madrid 520 rees per piastre. What mode of drawing ought a merchant of London to adopt, and what will he clear per cent. by a draft on one place and a remittance to the other?

Here 520 rees=33d. at Madrid.

33d.	52 ~~520~~ rees.
~~1000~~ ~~10~~	65
x	~~100~~

$$x=\frac{52\times65}{33}=102\tfrac{14}{33}\text{d. Ans.}$$

Hence $2\frac{14}{33}$ per cent. is gained by making a draft on Madrid.

Example 31.

Required the arbitrated rate of exchange between London and Lisbon, when Amsterdam gives 34s. 4d. Flemish per pound sterling, and 130d. Flemish per milree.

Pence sterling, ~~240~~	~~412~~ pence Flemish.
60	103
Pence Flemish, 130	x sterling.

$$\frac{60\times 130}{103}=75\tfrac{75}{103}\text{d.}=x, \text{ the Ans.}$$

Example 32.

If the course of exchange between London and Amsterdam be 11 florins, 14 stivers, between Amsterdam and Paris 53 groats for 3 francs, and between Paris and Madrid 15 francs for 1 pistole, what is the arbitrated exchange between London and Madrid?

£1	
Groats, 53	1 Pistole.
Francs, ~~15~~	
5	
Pistoles, x	
	468 groats = 11 flo. 14 stiv.
	~~3~~ Francs.

$$x=\tfrac{468}{265}=1\tfrac{203}{265} \text{ pistoles. Ans.}$$

Example 33.

Suppose London has a sum of money to receive at Cadiz, the exchange being at 38d.; but, instead of drawing directly on that place, he draws on Amsterdam, ordering his agent there to draw on Paris; and Paris to draw on Cadiz; the exchange between London and Amsterdam being 35s. Flemish per pound sterling; between Amsterdam and Paris $53\frac{1}{2}$ groats Flemish per 3 francs; and between Paris and Cadiz 15 francs, 50 centimes per doubloon of plate; what is the arbitrated price between London and Cadiz.

$$
\begin{array}{r|l}
\cancel{5} & \\
\cancel{20} & \\
\text{Sterling, pence, } \cancel{240} & \\
\text{Shillings, Flemish, } \dfrac{107}{\cancel{2}} & 2 \\
\text{Francs, } \dfrac{31}{\cancel{2}} & 2 \\
\hline
& 7\ \cancel{35}\ \text{shillings, Flemish.} \\
& 3\ \cancel{36}\ \text{francs.} \\
& 4\ \text{dollars.} \\
& \\
& \quad x \\
\end{array}
$$

$$\frac{107 \times 31}{4} = 39\frac{1}{2} = x. \text{ Ans.}$$

Example 34.

London purchases bills on Madrid at $33\frac{1}{2}$d. per dollar of exchange, remits them to Amsterdam to be negociated at $90\frac{1}{4}$ groats Flemish per ducat of exchange, and to make the returns in bills on Paris, at $51\frac{7}{8}$ groats per ecu of 3 francs, the bills on Paris are

negociated in London at 25 livres 18 sous per £ sterling, including interest, amounting to $1\frac{1}{8}$ per cent.; how much per cent. is gained or lost by this operation?

Arrangement and Operation.

Pence, $\frac{67}{\cancel{2}}$	$\cancel{34}$ 17
	$\cancel{272}$ Maravedes = 1 dol.
47	$\cancel{2}$
1 ducat = Mara. $\cancel{376}$	$\frac{361}{\cancel{4}}$ groats.
137	
Groats, $\frac{\cancel{411}}{\cancel{8}}$	3 francs.
Francs, $\cancel{80}$	81 livres.
	10
Livre, 1	$\cancel{20}$ sous.
Sous, $\cancel{518}$	$\cancel{3}$
259	$\cancel{240}$ pence.
Pence, $\cancel{100}$	$\frac{791}{\cancel{8}}$ pence.
x	$\cancel{103}$ pence.

$$x = \frac{17 \times 361 \times 3 \times 81 \times 10 \times 791}{67 \times 47 \times 137 \times 259} = \frac{11796111810}{111735967} =$$

105·579. Hence, 5·579 is gained per cent.

Example 35.

How many shares in the Anglo-Mexican Mining Association, at £85 premium, can I purchase for £50000 in the 3 per cent. consols, market price being at 90¼.

Operation.

x	
~~185~~	
37	
	500~~00~~
~~100~~	~~90~~
	18

$$\therefore x = \frac{500 \times 18}{37} = 243\tfrac{9}{37}. \text{ Ans.}$$

All the installments are supposed to be paid up.

Example 36.

How many bricks, 10½ inches long, 6½ inches broad, and 4½ inches thick, (including mortar,) will be wanted to build a garden wall 60 yards long, 10 feet high, and 2½ feet thick; and how much will they come to at £3 the thousand.

1st. *Operations.* 2nd.

7 ~~21~~	2
13	2
~~9~~	~~2~~
	~~60~~ 20
x	~~36~~ 4
	10
	12
~~2~~	5
	12

~~1000~~	3
x	
91	2304~~000~~

$$\therefore x = \frac{2304 \times 3}{91} = £75\ 19\ 1\tfrac{41}{91}. \text{ Ans.}$$

$$\therefore x = \frac{4 \times 20 \times 4 \times 10 \times 12 \times 5 \times 12}{7 \times 13} = \frac{2304000}{91} = 25318\tfrac{62}{91}. \text{ Ans.}$$

Example 37.

How many shares in the Northern Railroad, at £6 prem. can I purchase for 150 shares in the Anglo-Peruvian Mining Association, at £45 prem.

$$
\begin{array}{r|l}
x & \\
\cancel{106} & \\
53 & \\
\hline
 & \cancel{150}\ \ 75 \\
 & 145 \\
 & x=\dfrac{145\times 75}{53}=205\tfrac{10}{53}.\ \text{Ans.}
\end{array}
$$

Example 38.

How many brakes of tea, each brake 70 chests, each chest 80 lb. at 6s. per lb. can be bought for 170342 lire. 70 cents. Venice money, at 25 lire. 30 cents. per lb. sterling,

Operation.

$$
\begin{array}{r|l}
x & \\
\cancel{70} & \\
4\ \cancel{80} & \\
6 & \cancel{20} \\
\hline
25{\cdot}3 & \cancel{170342{\cdot}}\ \cancel{7} \\
 & 4258{\cdot}5675
\end{array}
$$

$$\therefore x=\frac{4258{\cdot}5675}{253\times 7\times 6}=4{\cdot}00769 \text{ brakes. Ans.}$$

Example 39.

How many shares in the great Northern Railroad, at £10 premium (£50 shares) can be purchased for 600 packs of broad cloth, each pack 50 pieces, each piece 30 yards, at 15s. per yard?

Operation.

$$\begin{array}{r|l}
x & \\
\cancel{60} & \\
\hline
 & \cancel{600} \quad \cancel{10} \\
 & \cancel{50} \quad 25 \\
 & 30 \\
\cancel{2} \quad \cancel{20} & 15 \\
 & \therefore x = 25 \times 30 \times 15 = 11250 \text{ shares. Ans.}
\end{array}$$

Example 40.

How many acres of land, at £150 per acre, can be purchased for £150000 in the 5 per cent. Navy Annuities, at $110\frac{1}{4}$?

Operation.

$$\begin{array}{r|l}
10 & \\
\cancel{150000} & \\
110 & \cancel{100} \\
\hline
 & x \\
 & \cancel{150} \\
 & \therefore x = 110 \times 10 = 1100 \text{ acres. Ans.}
\end{array}$$

Example 41.

How many brakes of tea, each 75 chests, each chest 85 lb. at 5s. the lb., can I purchase for 150 packs of broad-cloth, each pack 25 pieces, each piece 40 yards, at 25s. per yard?

$$
\begin{array}{rr|ll}
 & x & & \\
 & \cancel{75} & & \\
17 & \cancel{85} & & \\
 & \cancel{5} & & \\
\hline
 & & \cancel{150} & 2 \\
 & & \cancel{25} & 5 \\
 & & 40 & \\
 & & 25 & \\
\end{array}
$$

$$x = \frac{25 \times 40 \times 2}{17} =$$

117 bks. 48 ch. 45 lb. Ans.

Example 42.

Having £120000 in the 3 per cent. cons. which I wish to invest in the Anglo-Mexican Mining Association, the consols are at $85\frac{1}{4}$, and the shares at £110 prem., how many shares can I have?

Operation.

$$
\begin{array}{rr|l}
 & x & \\
7 & & \\
 & \cancel{2}.\cancel{0} & 40 \\
\hline
 & & \cancel{120000} \\
 & & 85 \\
 & \cancel{100} & \\
\end{array}
$$

$$x = \frac{40 \times 85}{7} = 485\tfrac{5}{7} \text{ shares.}$$ [Ans.

Example 43.

How many pipes of Madeira, at £35 a pipe, can I purchase for 150 shares in the British Mining-Association, at £40 per share prem.?

$$
\begin{array}{r|l}
x & \\
\cancel{7}\ \ \cancel{35} & \\
\hline
 & \cancel{150}\ \ 30 \\
 & \cancel{140}\ \ 20 \\
 & \\
 & x=30\times 20=600 \text{ pipes. Ans.}
\end{array}
$$

Example 44.

How many pipes of port, at £42 per pipe, can I purchase for £10000 in the $3\frac{1}{2}$ per cent. at $105\frac{1}{2}$.

$$
\begin{array}{r|l}
x & \\
\cancel{6}\ \ \cancel{42} & \\
\cancel{2}\quad\ \ & 50 \\
\hline
 & \cancel{10000} \\
\cancel{100} & \cancel{105} \\
 & \cancel{15} \\
 & 5 \\
 & \\
 & \therefore x=50\times 5=250 \text{ pipes. Ans.}
\end{array}
$$

Example 45.

If 15 men, in $\frac{1}{2}$ of $\frac{3}{4}$ of ten days, working $\frac{2}{3}$ of $\frac{3}{5}$ of 12 hours a day, dig a trench 500 yards long, 3 feet wide, and 5 feet deep; how many men in 30 days, working 10 hours a day, will dig a trench 7000 yards long, 4 feet wide, and 3 feet deep?

Operation.

Men, ~~15~~	
3	
1	~~2~~
~~3~~	~~4~~
~~5~~ ~~10~~	
2	~~3~~
~~3~~	~~5~~
~~8~~ ~~12~~	
	~~500~~ ~~100~~ 25
	~~3~~
	~~5~~
~~7000~~	x
14	~~30~~ ~~10~~
~~4~~	~~10~~
3	

$$\therefore x=\frac{3\times2\times14\times3}{25}=\frac{252}{25}$$
$$=10\tfrac{2}{25}.\text{ Ans.}$$

Example 46.

How many yards of drugget of $\frac{7}{4}$ wide, would cover a room 14 yards long and 8 yards wide?

Operation.

x	
~~7~~	4
	~~14~~ 2
	8

$x=8\times2\times4=64$ yds. Ans.

Example 47.

How many packs of cloth, each pack containing 45 pieces, each piece 25 yards, at 17s. 6d. per yard, for 125 brakes of tea, each brake 75 chests, each chest 80 lb. at 5s. per lb.?

Operation.

		x	
3	$\cancel{9}$	$\cancel{45}$	
		$\cancel{52}$	
	7	$\cancel{35}$	2
			$\cancel{125}$ 25
			$\cancel{75}$ $\cancel{25}$
			80
			$\cancel{5}$

$$x = \frac{25 \times 80 \times 2}{7 \times 3} = \frac{4000}{21}$$

$190\frac{10}{21}$. Ans.

Example 48.

A. can do a piece of work in 10 hours, B. can do it in 12 hours, C. in 8 hours, and D. in 14 hours; in what time would they do it, if they all worked together.

A. does the $\frac{1}{10}$ in an hour.
B. — $\frac{1}{12}$ in ditto.
C. — $\frac{1}{8}$ in ditto.
D. — $\frac{1}{14}$ in ditto.

Then $\frac{1}{10}+\frac{1}{12}+\frac{1}{8}+\frac{1}{14}=\frac{319}{840}$.

Operation.

$$\begin{array}{r|l} \begin{array}{c} 319 \\ \hline \cancel{840} \end{array} & 1 \text{ hour.} \\ \hline \text{hours } x & \begin{array}{c} 840 \\ \hline \cancel{840} \end{array} \\ & x = \dfrac{840}{319} = 2\text{ h } 37' \; 59\tfrac{199}{319}''. \text{ Ans.} \end{array}$$

Example 49.

In what time till £700 amount to £784, at 4 per cent. per annum?

Operation.

$$\begin{array}{r|l} \cancel{100} & \\ 1 & \cancel{4} \\ \hline \cancel{84} & \cancel{700} \\ \cancel{12} & \cancel{7} \\ 3 & x \end{array}$$

$\therefore x = 3$ years. Ans.

MENSURATION.

1. How much will a square piece of ground come to at 35s. per acre, each side of the square being 35 chains, 25 links.

square chains ~~10~~	1 acre.
acres x	~~35·25~~ 3·525
~~20~~	~~35·25~~ 1·7625
	35

$x = 3{\cdot}525 \times 1{\cdot}7625 \times 35 = £217\ 8\ 11{\cdot}625$. Ans.

The contents of a square is found by multiplying the side by itself.

As a chain contains 100 links, the links are decimals of a chain; in the above Example, 25 links $= \frac{25}{100}$ of a chain $= {\cdot}25{\cdot}$ 10 square chains make an acre.

Example 2.

How much would the paving of a triangular yard come to at 4s. the square yard, the base being 60 feet, and the perpendicular let fall upon it from the opposite angle 30 feet?

~~3~~	
~~3~~	4
x	~~60~~ ~~3~~
~~20~~	~~30~~ ~~10~~ 5
~~2~~	~~2~~

$x = 4 \times 5 = £20$. Ans.

The contents of a triangle is found by multiplying the base by ½ of the perpendicular. A square yard is 3 feet in length and 3 feet in breadth.

Example 3.

How much would the covering of a room with floor-cloth, at 5s. per yard, come to, the dimensions of the room being 40 feet long, 20 feet wide?

Operation.

$$\begin{array}{r|l} \text{long } 3 & \\ \text{broad } 3 & 1 \text{ yd.} \\ \hline x & 40 \text{ long.} \\ \cancel{20} & \cancel{20} \text{ broad.} \\ & 5 \end{array}$$

$$x = \frac{40 \times 5}{9} = \frac{200}{9} = £22 \quad 4 \quad 5\tfrac{1}{3}. \text{ Ans.}$$

Example 4.

How many tiles, 10 inches long, 4½ inches broad, will be required to pave a ball court 60 feet long and 25 feet wide?

Operation.

$$\begin{array}{r|l} \cancel{10} & \\ \cancel{3} & \\ 4\tfrac{1}{2} = \frac{\cancel{9}}{\cancel{2}} & \begin{array}{l} 1 \text{ brick.} \\ 2 \end{array} \\ \hline & \cancel{60} \quad \cancel{6} \quad 2 \\ x & \cancel{12} \quad 4 \\ & 25 \\ & 12 \end{array}$$

$$x = 25 \times 12 \times 4 \times 2 \times 2 = 4800.$$

In this operation the first cause is in inches, and the second cause in feet; we therefore multiply the 60 feet by 12, and the 25 feet by 12, to reduce them to the same denomination.

Example 5.

How many yards of carpeting of $\frac{3}{4}$ wide would be required to cover a floor 25 feet long and 18 feet broad?

Operation.

$$
\begin{array}{r|l}
x & \\
\frac{3}{4} & 4 \\
\hline
 & \cancel{18}\ \ \cancel{6}\ \ 2 \\
 & \overline{\cancel{3}} \\
\cancel{3} & \\
 & 25 \\
 & \overline{\cancel{3}} \\
\cancel{3} &
\end{array}
$$

$$x = \frac{25 \times 2 \times 4}{3} = \frac{200}{3} = 66\tfrac{2}{3}. \text{ Ans.}$$

In this example the first cause is in yards, we therefore divide the length and breadth of the floor by 3, to reduce them to yards.

Example 6.

How much will the paving of a hexagonal court-yard come to, at 3s. 6d. the square yard, each side being 28 feet, and the perpendicular from the centre on one of the sides 24 feet?

Operation.

$$
\begin{array}{r|l}
\cancel{3} & \cancel{42}\text{ d.} \\
\cancel{3} & \cancel{14}\ \ 7 \\
\hline
x & 28 \\
\cancel{240} & \ \cancel{6}\ \ \cancel{2} \\
\cancel{10} & \cancel{24} \\
5\ \ \cancel{2} & \overline{\cancel{2}}
\end{array}
$$

$$\therefore x = \frac{28 \times 7}{5} = \frac{196}{5} = £39\ \ 4. \text{ Ans.}$$

The area of a hexagon is found by multiplying the sum of the sides by half the perpendicular from the centre to the base.

Example 7.

How many square yards are in a circle whose diameter is $6\frac{1}{2}$ feet?

Operation.

3	
3	1 yard.
x	13
~~2~~	~~2~~
	13
~~2~~	~~2~~
	~~·7854~~
	·19635

$$x = \frac{\cdot 19635 \times 13 \times 13}{9} = 2\cdot 24257 \text{ yds. Ans.}$$

The area of a circle is equal to the square of the diameter multiplied by ·7854, or the square of the circumference multiplied by ·07958.

Example 8.

How much drugget would be required to cover a circular room, the diameter of which being 27 feet, and the drugget $\frac{3}{4}$ wide?

Operation.

x	
~~7~~	
~~4~~	
~~9~~	4
	~~27~~ 3
	27
	~~·7854~~
	·1122

$$x = \cdot 1122 \times 27 \times 3 \times 4 = 36\cdot 3528 \text{ yds. Ans.}$$

Example 9.

How much will the painting of a circular room come to, at 3s. 6d. the square yard, the diameter of the room being 30 feet and the height 15 feet.

Operation.

$$\begin{array}{r|l} \cancel{7} & \cancel{22} \\ \cancel{3}\ \ \cancel{9} & \quad 11 \\ \hline x & \cancel{30}\ \ \cancel{10} \\ \cancel{24}0 & \cancel{15}\ \ 5 \\ \cancel{4} & \cancel{42}\text{d.}\ \cancel{6} \\ 2 & \end{array}$$

$$x = \frac{11 \times 5}{2} = \frac{55}{2} = £27 \quad 10 \quad 0. \text{ Ans.}$$

In this example we multiply the first cause by 9, to bring the square yards into square feet, as the second cause produces square feet.

The diameter of a circle is to its circumference as 7 : 22.

Example 10.

If 10 pieces of plank, each 16 inches square at the base, and 12 feet long, weigh $\frac{3}{4}$ of a ton, how much will 80 pieces of plank come to, at $7\frac{1}{2}$d. per lb., each piece 18 inches square at the base, and 18 feet long?

Operation.

$\cancel{10}$	
$\cancel{16}$	$\cancel{15}$ 5
8 $\cancel{16}$	$\cancel{112}$
$\cancel{2}$ $\cancel{4}$ $\cancel{12}$	7
x	$\cancel{80}$ $\cancel{8}$ 4
4	$\cancel{18}$ 9
$\cancel{16}$	$\cancel{18}$ 9
$\cancel{80}$	$\cancel{18}$ $\cancel{9}$ 3
$\cancel{240}$	$\cancel{15}$
$\cancel{2}$	3

$$\therefore x = \frac{5\times7\times9\times9\times9}{4\times8} = \frac{25515}{32} =$$

£797 6 $10\frac{1}{2}$. Ans.

The solidity of a square plank is found by squaring the base, or end, and multiplying that by the length.

Example 11.

How many stones, 15 inches long, 12 inches broad, will be wanted to pave a yard 20 yards long, and 15 yards broad; and how much will the paving come to, at 4d. the square foot?

1st. *Operations.* 2nd.

$\cancel{15}$	1 stone.
$\cancel{12}$	
x	20
	$\cancel{36}$ 3
	$\cancel{15}$
	36

$\cancel{12}$	$\cancel{4}$
4 $\cancel{12}$	
$\cancel{12}$ $\cancel{240}$	
x	$\cancel{20}$
	$\cancel{36}$ $\cancel{3}$
	15
	$\cancel{36}$ 3

$\therefore x = 20\times3\times36 =$ 2160 stones. Ans. $\therefore x = 15\times3 =$ £45. Ans.

Example 12.

If 10 sugar-loaves, 15 inches diameter at the base, and 24 inches high, weigh 3 cwt. how much will 150 loaves of the same quality come to, each 18 inches diameter at the base, and 30 inches high, at 10d. per lb.?

Operation.

$$
\begin{array}{r|l}
\cancel{10} & \\
\cancel{15} & \\
\cancel{15} & \cancel{3} \\
\cancel{\cdot 7854} & \cancel{112} \\
8 \quad \cancel{24} & 14 \\
\cancel{3} & \cancel{8} \\
\hline
 & \cancel{150} \ \ \cancel{10} \\
 & \cancel{18} \quad 9 \\
 & \cancel{18} \quad 3 \\
\pounds x & \cancel{\cdot 7854} \\
 & \cancel{30} \quad \cancel{2} \\
\cancel{3} & \cancel{3} \\
\cancel{240} & \cancel{10} \\
\cancel{24} & \\
\cancel{12} & \\
\cancel{2} &
\end{array}
$$

$\therefore x = 14 \times 9 \times 3 = \pounds 378.$ Ans.

The solidity of a cone or pyramid is found by multiplying the area of the base into $\frac{1}{3}$ of the height.

Example 13.

How many acres of land at £250 per acre, can be purchased for £35000 in the 3 per cent. reduced, market price $94\frac{1}{8}$.

Operation.

$\cancel{250}$ 5	1 acre.
acres x	7
	$\cancel{35000}$
$\cancel{100}$	$\cancel{94 \cdot 125}$
	18·825 acres.

$\therefore x = 18 \cdot 825 \times 7 = 131 \cdot 775.$ Ans.

Example 14.

How many acres of land in a quadrilateral field, whose diagonal is 20 chains 46 links, and perpendiculars, let fall from the opposite angles on the diagonal, 5 chains 14 links, and 3 chains 20 links?

$$\left.\begin{array}{r} 5 \cdot 14 \\ 3 \cdot 20 \end{array}\right\} \text{perp.}$$

8·34 sum.

4·17 half sum.

Operation.

square chains $\cancel{10}$	1 acre.
acres x	$\cancel{0 \cdot 46}$ 2·046
	4·17

$\therefore x = 2 \cdot 046 \times 4 \cdot 17 = 8 \cdot 53182$ acres. Ans.

Example 15.

How many cubes whose side is 5 inches, can be made out of a globe of gold whose diameter is 30 inches.

Operation.

$$\begin{array}{r|l}
\cancel{5} & \\
\cancel{5} & 1 \text{ cube} \\
\cancel{5} & \\
\hline
\text{cubes } x & \cancel{30}\ 6 \\
 & \cancel{30}\ 6 \\
 & \cancel{30}\ 6 \\
 & \cdot 5236 \\
 & \\
 & \therefore x = \cdot 5236 \times 6 \times 6 \times 6 = \\
 & \qquad 113 \cdot 0976. \text{ Ans.}
\end{array}$$

The solidity of a cube is found by cubing the side; that of a globe by multiplying the cube of the diameter by ·5236.

Example 16.

How many bricks 9 inches long, $4\frac{1}{2}$ inches broad, and 3 inches thick, will build a wall 60 yards long, 10 feet high, and $2\frac{1}{2}$ feet thick?

Operation.

$$\begin{array}{r|l}
\cancel{9} & \\
\cancel{3}\ \cancel{9} & \cancel{2} \\
\cancel{3} & \quad 1 \text{ brick.} \\
\hline
x & \cancel{60}\quad 20 \\
 & \cancel{36}\quad 4 \\
 & 10 \\
 & \cancel{12}\quad 4 \\
\cancel{2} & \cancel{5} \\
 & \cancel{12}\quad 4
\end{array}$$

$$\therefore x = \overset{}{\underset{1}{\cancel{2}00}} \times 4 \times 5 \times 4 \times 4 = 64000 \text{ bricks.}$$

Example 17.

How many solid feet of wood are in 150 planks, each 10 inches broad, and 6 inches deep at the base, and 18 feet long?

Operation.

$$\begin{array}{r|l} \cancel{2}\ \ \cancel{12} & \\ \cancel{2}\ \ \cancel{12} & \\ \cancel{12} & 1 \text{ foot.} \\ \hline \text{feet } x & \cancel{10}\ \ 5 \\ & \cancel{6} \\ & \cancel{18}\ \ 3 \\ & \cancel{12} \\ & \cancel{150} \\ & 75 \end{array}$$

$$x = 75 \times 3 \times 5 = 1125$$

Example 18.

How many stones, 20 inches long, 15 inches broad, and 4 inches thick, will be required to build a square pyramid, each side of the base being 6 yards, and the perpendicular altitude 30 feet?

Arrangement and Operation.

$$\begin{array}{r|l} 5\ \ \cancel{20} & \\ \cancel{15} & 1 \text{ stone.} \\ \cancel{4} & \\ \hline \text{stones } x & \cancel{6} \\ & \cancel{36}\ \ 9 \\ & \cancel{6}\ \ 2 \\ & 36 \\ \cancel{3} & \cancel{30}\ \ 2 \\ & \cancel{12}\ \ 3 \end{array}$$

$$\therefore x = \frac{6 \times 9 \times 2 \times 36 \times 6}{5} = \frac{23328}{5} = 4665\tfrac{3}{5} \text{ stones.}$$

Ans.

The solidity of a square pyramid is found by squaring the side of the base, and multiplying that by $\frac{1}{3}$ of the altitude.

Example 19.

Required the price of a marble slab, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 8s. the square foot.

Operation.

12	
6 ~~12~~	1 foot.
3	
feet x	67
	~~22~~ 11
5	~~8~~ ~~2~~
~~20~~	

$$\therefore \frac{67\times 11}{12\times 5\times 3} = x = \frac{737}{180} =$$

£4 1 $10\frac{2}{3}$. Ans

Example 20.

To how much will the wainscoting of a room amount, at 6s. the square yard, the height being 12 feet 6 inches, and the whole compass 83 feet 8 inches ?

Operation.

9	
~~20~~	~~6s.~~
4	~~2~~
£x	
~~2~~	~~25~~ 5
~~3~~	251

$$\therefore x = \frac{251\times 5}{4\times 9} = £34\ 17\ 2\frac{2}{3}.$$

Ans.

Example 21.

A cable, that is 3 feet long and 9 inches circumference, weighs 22 lb., how much would a fathom weigh that is 12 inches circumference?

Operation.

~~3~~	
~~3~~ ~~9~~	
9	22 lb.
~~·07958~~	
lb. x	~~6~~ 2
	~~12~~ 4
	~~12~~ 4
	~~·07958~~

$$x = \frac{22 \times 32}{9} = 78\tfrac{2}{9} \text{ lb. Ans.}$$

The solidity is found by multiplying the area of the base into the length.

Example 22.

If 20 iron railings, 6 feet high, weigh half a ton, when the bars are $1\frac{1}{4}$ inch square, what will 50 of the same height come to at $3\frac{1}{2}$d. per lb. the base being $\frac{7}{8}$ of an inch square?

Operation.

~~2~~ ~~20~~	
~~6~~	
~~5~~	~~4~~ ~~2~~
~~5~~	~~4~~
	~~10~~
	~~112~~ ~~14~~ 7
x	~~6~~
~~4~~ ~~8~~	~~50~~ ~~10~~ ~~2~~
~~8~~	7
~~2~~	7
~~240~~	7
120	

$$\therefore x = \frac{7 \times 7 \times 7 \times 7}{120} = \frac{2401}{120} = £20 \quad 0 \quad 2. \text{ Ans.}$$

Example 23.

How many bricks would be required to build a wall 10 feet high, 500 feet long, and a brick and half thick; the brick being 10 inches long, and 4 courses to the foot in height.

Operation.

$$
\begin{array}{r|ll}
\cancel{10} & & \\
\cancel{3} & 1 & \\
\cancel{5} & & \\
\hline
x & \cancel{500} & 50 \\
 & 12 & \\
 & 10 & \\
 & 12 & \\
 & \cancel{15} & 3
\end{array}
$$

$\therefore x = 50 \times 10 \times 12 \times 12 = 72000$ bricks. Ans.

See Hutton, page 84, vol. ii.

Example 24.

How many bricks would be required to build a square pyramid, each side of the base being 100 feet, and perpendicular height being 100 feet, the dimensions of the brick being 10 inches long, 5 inches broad, 3 inches thick?

Operation.

$$
\begin{array}{r|l}
\cancel{10} & \\
\cancel{5} & \\
\cancel{3} & 1 \text{ brick.} \\
\hline
x & \cancel{100}\ \cancel{10}\ 2 \\
 & \cancel{12}\ 4 \\
 & 100 \\
 & 12 \\
3 & 100 \\
 & \cancel{12}\quad 4
\end{array}
$$

$\therefore x = 100 \times 100 \times 12 \times 32 = 3840000$ bricks. Ans.

As a pyramid is equal to $\frac{1}{3}$ of a prism of the same base and altitude, $\frac{1}{3}$ of the altitude drawn into the area of the base is the solidity.

Example 25.

A cubic foot of brass is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter, what will the length of the wire be, allowing no loss of metal?

Operation.

$$\begin{array}{r|l}
12 & \\
4\ \cancel{12} & \\
1\text{ cubic foot.}\qquad 2\ \cancel{12} & \\
\hline
40 & 1 \\
40 & 1 \\
 & \cancel{7854}\ \ \cdot 3927 \\
 & \cancel{36}\ \ 3 \\
 & x
\end{array}$$

$$\therefore x = \frac{40 \times 40 \times 12 \times 2}{\cdot 3927} = 97784\cdot 797 \text{ yards. Ans.}$$

Example 26.

A roof which is 24 feet 8 inches by 14 feet 6 inches, is to be covered with lead which weighs 8 lb. the square foot: what will it come to, at 18s. per cwt.?

Operation.

$$\begin{array}{r|l}
14 & \\
\cancel{112} & \cancel{18}\ \ 6 \\
10\ \ \cancel{20} & \quad 3 \\
\hline
\cancel{8} & \cancel{74}\ \ 37 \\
\cancel{2} & 29 \\
x & \cancel{8}
\end{array}$$

$$x = \frac{37 \times 29 \times 3}{14 \times 10} = \frac{3219}{140} = £22\ \ 19\ \ 10\tfrac{2}{7}. \text{ Ans.}$$

Example 27.

How much will the carpeting of an elliptical room come to, at $5\frac{1}{2}$s. the square yard, the axes of the ellipsis being 55, and 42 feet?

Square yard.	$\cancel{3}$ $\cancel{3}$ $\cancel{2}$	$5\frac{1}{2}=\frac{11}{\cancel{2}}$
	x	$\cancel{55}$ 11
		$\cancel{42}$ $\cancel{14}$ 7
	$\cancel{20}$	$\cancel{\cdot 7854}$
	$\cancel{4}$	$\cancel{\cdot 2618}$
	2	·1309

$$x=\frac{\cdot 1309\times 11\times 11\times 7}{2}=£55\cdot 43615.$$

The contents of an ellipsis is found by multiplying the longer diameter by the shorter, and multiplying the product by ·7854.

Example 28.

How many stones, 14 inches long and 10 inches wide, would be required to pave an elliptical court-yard, the axes being 24 and 18 yards?

$\cancel{7}$ $\cancel{14}$	
$\cancel{5}$ $\cancel{10}$	1 stone.
	$\cancel{24}$ $\cancel{12}$ 6
x	$\cancel{36}$ 7·2
	18
	36
	·1122

$$x=\cdot 1122\times 36\times 18\times 7\cdot 2\times 6=3140\cdot 88192 \text{ stones. Ans.}$$

Example 29.

How many yards in a parabola whose height is 15 feet, and base 24?

$\cancel{3}$	1 yard.	
$\cancel{3}$		
yards x	$\cancel{15}$	5
3	$\cancel{24}$	8
	$\frac{2}{\cancel{3}}$	

$$x=\frac{5\times 8\times 2}{3}=\frac{80}{3}=26\tfrac{2}{3}\text{ yds.}\quad \text{Ans.}$$

A parabola is $\frac{2}{3}$ of a parallelogram, having the same base and altitude, therefore $\frac{2}{3}$ of the product of the height and the base equal the superficial content.

Example 30.

Find the solidity, in feet, of a cube whose side is 28 inches?

One cubic foot.	3	$\cancel{12}$	1 foot.	
	3	$\cancel{12}$		
		$\cancel{12}$		
		3		
		x	$\cancel{28}$	7
			$\cancel{28}$	7
			$\cancel{28}$	7

$$x=\frac{7\times 7\times 7}{3\times 3\times 3}=\frac{343}{27}=12\tfrac{19}{27}.\ \text{Ans.}$$

The solidity of a cube equals the cube of the side.

Example 31.

How many solid feet are in a plank whose length is 5 feet 6 inches, and each side of the base 10 inches.

$\cancel{2}$ $\cancel{12}$	
6 $\cancel{12}$	1
12	
x	$\cancel{10}$ 5
	$\cancel{10}$ 5
	$\cancel{66}$ 11

$$x=\frac{11\times 25}{6\times 12}=\frac{275}{72}=3\tfrac{59}{72}\text{ feet. Ans.}$$

Example 32.

Bartered a cylindrical vessel of British spirits at 12s. 6d. per gallon, for tea at 5s. per lb.; the diameter of the vessel is 4 feet 6 inches, the depth 3 feet 4 inches, and 277·27 solid inches contain 1 gallon imperial measure; how much tea am I to receive?

277·27	$\cancel{12\tfrac{1}{2}}=\frac{\cancel{25}}{\cancel{2}}$ 5
$\cancel{2}$	
	54
x	54
	·7854
$\cancel{5}$	$\cancel{40}$ 20

$$x=\frac{54\times 54\times \cdot 7854\times 20\times 5}{277\cdot 27}=825\cdot 991\text{ lb. Ans.}$$

The solidity of a cylinder is equal to the area of the base drawn into the height.

Example 33.

If a square pyramid have each side of the base 18 inches, and the height 30 inches, how many solid feet does it contain?

$$
\begin{array}{r|l}
2\ \cancel{12} & \\
2\ \cancel{12} & 1\text{ foot.} \\
2\ \cancel{12} & \\
\hline
\text{feet } x & \cancel{18}\ \ \cancel{3} \\
 & \cancel{18}\ \ 3 \\
\cancel{3} & \cancel{30}\ \ 5
\end{array}
$$

$x=\frac{15}{8}=1\frac{7}{8}$. Ans.

Example 34.

How many stones 16 inches long, 12 inches broad, and 8 inches thick, would be required to build a square pyramid, each side of the base being 20 feet and the perpendicular altitude 45 feet?

$$
\begin{array}{r|l}
\cancel{2}\ \ \cancel{4}\ \ \cancel{16} & \\
\cancel{2}\ \ \cancel{8} & 1\text{ stone.} \\
\cancel{12} & \\
\hline
 & \cancel{20}\ \ 5 \\
\text{stones } x & \cancel{12}\ \ 3 \\
 & \cancel{20}\ \ \cancel{10}\ \ 5 \\
 & \cancel{12} \\
\cancel{3} & \cancel{45}\ \ 15 \\
 & \cancel{3} \\
 & \cancel{12}\ \ 6
\end{array}
$$

$x=5\times3\times5\times15\times6=6750$ stones. Ans.

Example 35.

If 40 solid feet of iron railing weigh a ton, when the base is $1\frac{3}{4}$ inches square, how much will 50 of the same length come to at $3\frac{1}{2}$d. per lb. when the base is $\frac{7}{8}$ of an inch square?

	~~56~~ 7
~~40~~	~~2240~~
~~7~~	~~4~~
~~7~~	~~4~~
x	~~50~~
2 ~~8~~	~~7~~
2 ~~8~~	~~7~~
2	7
3 ~~240~~	

$$x=\frac{7\times5\times7}{8\times3}=£10\quad 5\quad 2.\ \text{Ans.}$$

Example 36.

How many 3-inch cubes may be cut out of a 12-inch cube?

~~3~~	
~~3~~	
~~3~~	
x	~~12~~ 4
	~~12~~ 4
	~~12~~ 4

$$\therefore x=4\times4\times4=64.\ \text{Ans.}$$

Example 37.

How much would the gilding of the ball on the top of St. Paul's come to, at $3\frac{1}{2}$d. the square inch, the diameter of the ball being 6 feet?

1			
~~2~~	7		
~~240~~			
~~20~~			
5	~~72~~	6	
x	~~72~~	~~18~~	9
	3·1416		

$$\therefore x = \frac{3\cdot1416 \times 54 \times 7}{5} = £237\cdot50496. \text{ Ans.}$$

Example 38.

A schoolmaster has a play-ground 350 feet long and 200 feet broad, which he wishes to raise $1\frac{1}{2}$ feet by gravel dug out of a ditch that surrounds it; now if the ditch be 10 feet wide, how deep must it be dug?

$350 + 20 = 370 \times 2 = 740 =$ length of both sides of the ditch. $200 \times 2 = 400 =$ length of both ends.

$\overline{}$
1140

	x		
~~57~~	~~1140~~		
19	~~10~~		
		~~350~~	17
		~~200~~	~~20~~
	~~2~~	~~3~~	

$x = \frac{178}{19} = 9\frac{4}{19}$ feet. Ans.

Example 39.

How many pieces of paper, each piece 63 feet, would be required to paper a room 26 feet 6 inches long, 18 feet 3 inches broad, and 12 feet 9 inches high?

$$2\left(26\tfrac{1}{2}+18\tfrac{1}{4}\right)=89\tfrac{1}{2}=\text{circuit.}$$

Operation.

~~63~~	1 piece.
21	
2	~~$89\frac{1}{2}$~~ $=179$ the circuit.
4 x	~~$12\frac{3}{4}$~~ $=$ ~~51~~ 17

$$x=\frac{179\times 17}{21\times 2\times 4}=18\tfrac{19}{168}\text{ pieces. Ans.}$$

Example 40.

How much would the lining of a rectangular cistern with lead, come to, at 4d. per lb., the dimensions of the cistern being 7 feet 6 inches long, 3 feet 8 inches broad, and 3 feet 4 inches deep, if 3 square feet of surface weigh 22 lb.?

5 ~~15~~	~~2~~
10	3
~~2~~	

50 feet the area of the two sides.

$7\frac{1}{2}+(3\frac{1}{3}\times 2)\times 3\frac{2}{3}=14\cdot 6\times 3\frac{2}{3}=\frac{935}{18}$ the area of the bottom, and both ends.

$\therefore\ 50+\frac{935}{18}=\frac{1835}{18}$ area of the inside of the cistern.

Then,

$$\begin{array}{r|l}
1\,\text{lb.} & \cancel{4}\text{d.} \\
\hline
x & \cancel{1835} \\
18 & 367 \\
\cancel{60}\ \cancel{240} & \\
\cancel{12}\quad 3 & \cancel{22} \\
6 & 11
\end{array}$$

$$x = \frac{367 \times 11}{18 \times 18} = £21\ \ 14\ \ 4\tfrac{1}{2}.\ \text{Ans.}$$

Example 41.

If a round pillar, 10 inches diameter, be 60 feet, how many inches must be the diameter of a pillar of equal height, containing 90 feet?

$$\begin{array}{r|l}
\cancel{60} & 10^2 = \cancel{100} \\
\hline
x^2 & \cancel{90}\ 15 \\
 & x^2 = 15 \times 10 = 150 \\
 & x = \sqrt{150} = 12{\cdot}246.\ \text{Ans.}
\end{array}$$

Cylinders and cones of equal altitude are to one another as the squares of their diameters.—EUC. Prop. xi. Book xii.

Example 42.

How much will the paving of a foot-path cost, at 3s. 4d. the yard, the length being 45 feet, and the breadth 4 feet 8 inches?

$\cancel{9}$	$\cancel{40}$
	$\cancel{45}$ 5
3	$\cancel{4\frac{2}{3}} = \cancel{14}$
x	7
$\cancel{240}$	
3 $\cancel{6}$	

$x = \frac{5 \times 7}{9} = \frac{35}{9}$. £3 17 $8\frac{2}{3}$. Ans.

Example 43.

How much will the covering of a roof with lead come to, at 16s. per cwt. the roof being 39 feet 4 inches long, and 3 feet 9 inches broad, if $8\frac{1}{3}$ lb. equal 1 square foot?

$\cancel{112}$	$\cancel{16}$ 4
56	
x	$\cancel{39\frac{1}{3}} = \cancel{118}$ 59
3	$\cancel{3\frac{3}{4}} = \cancel{15}$ 5
$\cancel{4}$	$\cancel{8\frac{1}{3}} = \cancel{25}$ 5
$\cancel{3}$	
4 $\cancel{20}$	

$x = \frac{59 \times 5 \times 5}{56 \times 3} =$ £8 15 $7\frac{1}{7}$. Ans.

Example 44.

To find the area of a sector, whose circumference is 18°, and diameter 3 feet?

Operation.

$$\begin{array}{r|l} \cancel{360} & \cancel{18} \\ \cancel{20} & \\ \hline & 3 \\ x & \cancel{3}\cdot 3 \\ & \cancel{\cdot 7854} \\ & \cdot 3927 \end{array}$$

$\therefore x = \cdot 3927 \times 3 \times \cdot 3 = \cdot 35343$ feet. Ans.

See Hutton, vol. ii. page 41.

To find the area of a sector, calculate the area of the circle from the diameter; then say, as 360 are to the number of degrees in the circle, so is the area found to that of the sector.

The following is the operation by Rule 1 in the course.

$$\begin{array}{r} 3\cdot 1416 \\ 3 \\ \hline 9\cdot 4248 \\ \hline\hline \end{array}$$

and 360 : 18 :: 9·4248

$$\begin{array}{rr|l} & 9\cdot 4248 & \\ & 18 & \\ \cline{2-2} & 753984 & \\ & 94248 & \\ \cline{2-2} 360 & 169\cdot 6464 & \cdot 47124 \\ & 1440 & \\ \cline{2-2} & 2564 & \\ & 2520 & \\ \cline{2-2} & 446 & \\ & 360 & \\ \cline{2-2} & 864 & \\ & 720 & \\ \cline{2-2} & 1440 & \\ & 1440 & \\ \end{array}$$

```
then ·47124
          3 diameter.
  4 | 1·41372
      ·35343 Ans. as above.
```

Operation by Rule 2.

```
·7854
    9 square of diameter.
70686
```

```
Thus, 360  :  18  ::  7·0686
                          18
                      565488
                      70686
                360 | 127·2348 | ·35343
                      1080
                       1923
                       1800
                        1234
                        1080
                         1548
                         1440
                          1080
                          1080
```

QUESTIONS FOR PRACTICE ON MENSURATION.

1. How many stones, 15 inches long, 6 inches broad, and 4 inches thick, would be required to build 10 round pillars, 20 feet high, and 4 feet diameter?
Ans. 12063·744 stones.

2. How much would the bricks cost, at £3 15s. per thousand, to build a cone 15 feet in circumference, and 30 feet high; the dimensions of a brick being 10 inches long, $4\frac{1}{2}$ inches broad, $2\frac{1}{2}$ inches thick?
Ans. £10 6 3·25632.

This question is reduced to multiplication by cancelling.

3. How many acres are in a piece of land in form of a rhombus, whose length is 10·34 chains, and breadth 6·25? Ans. 6 acres, 1 rood, 34 poles.

4. To find the number of square yards in a rhomboid, whose length is 36 feet, and height 6 feet 3 inches? Ans. 25 yards.

5. How many yards are in a circle whose diameter is 15 feet? Ans. 19·635.

6. If a cone, whose diameter is 12 inches, be 6 solid feet, how much would be the diameter of a cone of the same height, whose solidity is 10 times the other?
Ans. 37·94 inches.

7. How many square feet are in a circle, whose diameter is 40 inches? Ans. $6·9813\frac{1}{3}$ square feet.

8. How many square feet are in 150 boards, whose length are 12 feet, 7 inches, and breadth 9 inches?
Ans. $1415\frac{5}{8}$.

ADDITIONAL QUESTIONS.

QUESTIONS TAKEN FROM DALBY'S MATHEMATICS, PAGE 104, VOL. I.

Question 1.

If 12 persons spend £80 in 4 months, how much will 32 persons spend in 8 months?

Operation.

$$\begin{array}{r|l} 3\ \ \cancel{12} & 80 \\ \cancel{4} & \\ \hline x & \cancel{32}\ \ 8 \\ & \cancel{8}\ \ 2 \end{array}$$

$$\therefore x = \frac{80 \times 16}{3} = \frac{1280}{3} = £426\ \ 13\ \ 4.\ \ \text{Ans.}$$

DALBY'S FIRST METHOD.

1*st*. *By two Operations.*

Here the statings are both direct, for more persons will require more money, and more months will require more money.

$$\text{Hence } \overset{\text{per.}}{12} : \overset{\text{per.}}{32} :: \overset{£}{80} : \frac{80 \times 32}{12} = \frac{640}{3} =$$

the money that 32 persons would spend in 4 months.

$$\text{Again } \overset{\text{m.}}{4} : \overset{\text{m.}}{8} :: \overset{£}{\frac{640}{3}} : \frac{8 \times 640}{4 \times 3} = \frac{1280}{3} =$$

£426 13 4, the Ans.

2d. By one Operation.

$$\begin{array}{rcrcl} 12 & : & 32 & :: & 80 \text{ direct.} \\ 4 & : & 8 & :: & \text{— direct.} \end{array}$$

$$\frac{32\times8\times80}{12\times4} = \frac{1280}{3} = £426\ \ 13\ \ 4 \text{ the Ans. as before.}$$

$$\text{Or thus—As } \begin{array}{rcrcl} 12 & : & 80 & :: & 32 \text{ direct.} \\ 4 & : & & & 8 \text{ direct.} \end{array}$$

$$\text{Therefore } \frac{80\times32\times8}{12\times4} = \frac{1280}{3} = £426\ \ 13\ \ 4. \text{ Ans.}$$

Question 2.

If 1000 men besieged in a town with provisions for 28 days, at 18 oz. per day, be reinforced with 600 men, supposing they could not be relieved till the end of 42 days, how many ounces a day must each man have, that the provisions may last the time?

Operation.

$$\begin{array}{r|l} 5\ \cancel{1000} & \\ 2\ \cancel{28} & \\ 3\ \cancel{6}\ \cancel{18} & \\ \hline & \cancel{8}\ \cancel{4}\ 2 \\ & \cancel{1600} \\ & \cancel{42}\ \cancel{8} \\ & x \end{array}$$

$$\therefore x = \frac{5\times3}{2} = 7\tfrac{1}{2} \text{ oz. Ans.}$$

DALBY'S METHOD.

$1000 \times 18 \times 28$ ounces the whole quantity of provisions. This quantity is to last 1600 men 42 days, divide by 1600, and we have $\frac{1000 \times 18 \times 28}{1600}$ oz. the quantity which must last 1 man 42 days; this divided by 42, will give the allowance per day for 1 man, viz. $\frac{1000 \times 18 \times 28}{1600 \times 42} = \frac{10 \times 18 \times 28}{16 \times 42} = \frac{10 \times 9 \times 2}{8 \times 3} = \frac{5 \times 3}{2} =$ $7\frac{1}{2}$ oz. the answer.

Question 3.

If a garrison of 300 men have provisions for 5 weeks, at 12 ounces a day, how many men will the same quantity maintain 10 weeks, if each man is limited to 8 oz. per day?

Operation.

$$\begin{array}{r|l} \cancel{30}\ \ \cancel{300} & \\ 15\ \ \ \ \ \ & \\ 5 & \\ 3\ \ \cancel{12} & \\ \hline & x \\ & \cancel{10} \\ & \cancel{8}\ 2 \end{array}$$

$x = 15 \times 15 = 225.$

DALBY'S METHOD.

In this example both statings are inverse, and 10 and 8 are the terms of demand. Hence 10 w. : 5 w. :: 300 m. : $\frac{300 \times 5}{10} = 150$ the number of men the pro-

visions would serve 10 weeks at the rate of 12 ounces per day.

And 8 oz. : 12 oz. :: 150 m. : $\frac{12\times150}{8}=\frac{3\times150}{2}=225$ men, the answer.

The same, by one Operation.

First way. 10 : 5 :: 300 inverse.
8 : 12 inverse.

$$\frac{5\times12\times300}{10\times8}=225 \text{ the same as before.}$$

Second way. 5 : 300 :: 10 inverse.
12 8

$$\frac{300\times5\times12}{10\times8}=225, \text{ the answer.}$$

Question 4.

If a company of 160 men in 6 days of 11 hours each, can dig a trench 230 yards long, $5\frac{1}{2}$ wide, and $1\frac{1}{2}$ deep, in how many days, of 8 hours long, would another company consisting of 96 men, dig a trench 220 yards long, $3\frac{1}{2}$ wide, and 1 deep; supposing the hardness of the ground in the former case is to that in the latter as 5 to 7, and that 4 men of the latter company can do as much work as 5 of the former in the same time?

Operation.

$\cancel{160}$	$23\cancel{0}$
$\cancel{2}$ 6	$\cancel{11}$
$\cancel{2}$ $\cancel{11}$	$\cancel{3}$
$\cancel{4}$	5
44 $\cancel{220}$	x
7	$\cancel{8}$ $\cancel{2}$
1	$\cancel{96}$ $\cancel{6}$
7	5

$$\therefore x=\frac{44\times49}{23\times15}=6\tfrac{86}{345}. \text{ Ans.}$$

DALBY'S METHOD.

$230 \times 5\frac{1}{2} \times 1\frac{1}{2} = 1897\frac{1}{2}$ (by Mensuration) cubic yards in the first trench.

$220 \times 3\frac{1}{2} \times 1 = 770$ cubic yards in the other.

Now if we suppose the labour necessary to raise a like quantity of earth to be directly proportional to hardness of the required, it is evident that the strength required to dig the former trench, will be to that required for the latter, as $1897\frac{1}{2} \times 5$ to 770×7. And as 4 m. : 5 m. :: 96 : 120 m.; therefore, the labour of 120 men of the first company is equal to that of the 96 men.

Hence the question is reduced to the following.—If 160 men in 66 hours (6×11) can dig $1897\frac{1}{2}$ yards, in what time would 120 men dig 770×7?

As 160 m. : $1897\frac{1}{2} \times 5$ yds. :: 120 m. : $\dfrac{1897\frac{1}{2} \times 5 \times 120}{160}$ the yards which 120 men could dig in 66 hours. As $\dfrac{1897\frac{1}{2} \times 5 \times 120}{160} : 66 :: 770 \times 7 : \dfrac{66 \times 775 \times 7 \times 160}{1897\frac{1}{2} \times 5 \times 120}$ $= \dfrac{22 \times 154 \times 7 \times 8}{3795}$ hours, which, divided by 8, gives $6\frac{946}{3795}$ days, the answer.

By a single Operation.

First way.	96 m.	:	160 m.	::	6d. inverse.
	8 h.	:	11 h.		inverse.
	230 £	:	220		
	$5\frac{1}{2}$ br.	:	$3\frac{1}{2}$ br.		
	$1\frac{1}{2}$ d.	:	1 d.		
	5 har.	:	7 har.		
	5 m.	:	4 m.		

$\dfrac{160 \times 11 \times 20 \times 3\frac{1}{2} \times 1 \times 7 \times 4 \times 6}{96 \times 8 \times 230 \times 5\frac{1}{2} \times 1\frac{1}{2} \times 5 \times 5} =$, when reduced to the lowest terms, $\frac{24716}{3795} = 6\frac{946}{3795}$ days, the Answer.

Second way.	160 m.	:	6d.	::	96 m.	inverse.
	11 h.	:		::	8 h.	inverse.
	£230	:		::	220	
	$5\frac{1}{2}$ br.	:		::	$3\frac{1}{2}$ br.	
	$1\frac{1}{2}$	:		::	1d.	
	5 har.	:		::	7 har.	
	5 m.	:		::	4 m.	

$$\frac{6\times160\times11\times220\times3\frac{1}{2}\times1\times7\times4}{96\times8\times230\times5\frac{1}{2}\times1\frac{1}{2}\times5\times5}=\frac{24716}{3795}\text{, as before.}$$

INVERSE PROPORTION.—PAGE 102, VOL. I.

Question 1.

A garrison of 488 men have provisions for 39 weeks; how long would these provisions last, if the garrison be increased to 782?

Operation.

~~244~~ ~~488~~		
2 ~~39~~		
13		
		x
		~~782~~
		~~366~~
		~~122~~

$2\times13=26=x$. Ans.

DALBY'S METHOD.

$$732 : 488 :: 39 : \frac{488 \times 39}{732}$$

$$\begin{array}{r}
488 \\
39 \\
\hline
4392 \\
1464 \\
\hline
732 \,)\, 19032 \,(26 \\
1464 \\
\hline
4392 \\
4392 \\
\hline
\cdots
\end{array}$$

$$\text{Or,} \quad 488 : 39 :: 732 : \frac{488 \times 39}{732} = 26$$

$$\therefore 488 : 39 :: 732 : 26$$

$\therefore 488 \times 26 = 732 \times 39$, an absurdity, (see ar. 96, p. 97.)

Question 5.

A garrison of 800 men have provisions for 7 months; how long will these provisions last, if the garrison be increased to 1260 men?

Operation.

$$\begin{array}{rr|l}
40 & \cancel{800} & \\
 & \cancel{7} & \\
\hline
 & & x \\
 & & \cancel{1260} \\
 & & \cancel{18} \\
 & & 9
\end{array}$$

$$x = \tfrac{40}{9} = 4\tfrac{4}{9} \text{ Ans.}$$

DALBY'S METHOD.

$$1260 : 800 :: 7 : \frac{800 \times 7}{1260} = \frac{40 \times 7}{63} = \frac{40}{9} = 4\tfrac{4}{9}. \text{ Ans.}$$

MECHANICAL POWERS.

Mechanical Powers are certain simple instruments commonly employed for raising great weight, or overcoming great resistance. They are generally ranged under six heads; viz. the lever, wheel and axle, pulley, inclined plane, wedge, and screw.

OF THE LEVER.

Lever is an inflexible beam, which is used to raise weights, and is supported by a prop or fulcrum which is the centre of motion about which the body moves.

In all mechanical powers, power may be represented by weight.

When weight and power keep the lever in equilibrio, the weight is to the power reciprocally as their distance from the centre of motion. Or the distance of the power from the centre of motion multiplied into the power is equal to the weight multiplied into its distance from the centre of motion. Or the power and its distance may be considered the first cause, and the distance and weight the second cause, both producing the same effect.

Example 1.

What weight being placed 12 feet from the fulcrum, would balance 15 tons placed 4 feet on the other side of it?

Operation.

$$\begin{array}{r|ll} x & & \\ \cancel{12} & & \\ \cancel{3} & & \\ \hline & \cancel{15} & 5 \\ & \cancel{4} & \end{array}$$

$x = 5$ tons. Ans.

Example 2.

Suppose I have $1\frac{1}{2}$ tons of goods to raise by a lever: now if the fulcrum be placed 2 feet from the end on which the weight is to be raised, how long must the other end of the lever be, so that I can keep the $1\frac{1}{2}$ tons in equilibrio by 120 lb.?

$$
\begin{array}{r|l}
\text{cwt.} & \\
\cancel{30} & \\
28\ \ \cancel{112} & \\
2 & \\
\hline
 & \cancel{120}\ \ \cancel{4} \\
 & x
\end{array}
$$

$\therefore x = 28 \times 2 = 56$ feet. Ans.

PROOF.

$$
\begin{array}{r|l}
\cancel{30} & \\
\cancel{56}\ \ \cancel{112} & \\
\cancel{2} & \\
\hline
 & \cancel{120}\ \ \cancel{4}\ \cancel{2} \\
 & \cancel{56}
\end{array}
$$

Example 3.

What weight will $1\frac{1}{2}$ cwt. raise, being placed at the end of a handspike 100 inches long, the fulcrum being $7\frac{1}{2}$ inches from the other end?

$$
\begin{array}{r|l}
\text{cwt.} \qquad\qquad\qquad 56 & \\
1\frac{1}{2} \quad = \quad \text{lb. } \cancel{168} & \\
100-7\frac{1}{2} = \cancel{92\frac{1}{2}} & \\
\cancel{185} & \cancel{2} \\
37 & \\
\hline
\cancel{2} & \cancel{15}\ \ \cancel{3} \\
 & x
\end{array}
$$

$\therefore x = 56 \times 37 = 2072$ lb. Ans.

Example 4.

What weight placed on a steel-yard at 70 inches from the centre of motion will keep $9\frac{1}{2}$ cwt. in equilibrio at 2 inches distance from it on the other side?

$$\begin{array}{r|l} \cancel{112} & \\ 8 \quad 19 & \cancel{2} \\ \cancel{2} & \\ \hline & x \\ & \cancel{70} \\ & 5 \end{array}$$

$$\therefore x = \frac{19 \times 8}{5} = \frac{152}{5} = 30\tfrac{2}{5}. \text{ Ans.}$$

Example 5.

If 500 lb. be applied to the head of a rectangular wedge, whose thickness is 2 inches and length of its sides 12 inches, what weight will it balance perpendicular to its sides?

$$\begin{array}{c|c} \cancel{2} & 500 \\ \hline x & \cancel{12}\ 6 \end{array}$$

$$500 \times 6 = x = 3000. \text{ Ans.}$$

See Hutton, vol. ii. p. 159.

Example 6.

Two men carrying a load of two hundred weight between them on a pole 8 feet long, which rested on their shoulders, the weight hanging one foot from the centre; how much of the weight pressed on each man?

$$\begin{array}{c|l} \cancel{8} & \cancel{224}\text{ lb.} \\ & 28 \\ \hline x & 5 \\ & \qquad x = 28 \times 5 = 140\text{lb. then} \\ & 224 - 140 = 84\text{lb. the other.} \end{array}$$

OF THE WHEEL AND AXLE.

The weight and power are in equilibrio when the power is to the weight as the radius of the axle is to that of the wheel; or, in other words, the power multiplied into the radius of the wheel, equal the weight multiplied into the radius of the axle. This is evidently a lever in which the fulcrum is the common centre; hence, as in the lever, the power and the distance from the centre of motion may be considered the first cause. The weight, and its distance from the same centre, the second cause, both producing the same effect.

Example 7.

What weight will 3cwt. keep in equilibrio attached to a wheel whose radius is 3 feet, the weight being attached to an axle, whose radius is 8 inches?

Operation.

$$\begin{array}{r|l} \text{cwt. } 3 & \\ \text{inches } \cancel{36}\ 9 & \\ \hline & x \\ & \cancel{8}\ 2 \end{array} \qquad \text{Or,} \qquad \begin{array}{c|c} 3 & x \\ \hline \cancel{36} & \cancel{8}\ 2 \\ 9 & \end{array}$$

$$\therefore x = \frac{3 \times 9}{2} = 13\tfrac{1}{2} \text{ cwt.} \qquad \therefore \frac{9 \times 3}{2} = 13\tfrac{1}{2} \text{ cwt.}$$

Example 8.

Suppose a system of wheels and axles act upon each other, the radii of wheels being 6, 9, 9, 10, 12, 18 inches, and the radii of the axles 3, 3, 3, 3, 3, 3; how many pounds weight must be applied to the first

wheel to keep 1440 lb. attached to the axle of the last wheel, the friction not being considered?

The velocity produced would be 1440 : 1

$$
\begin{array}{r|ll}
\cancel{2} & & \\
\cancel{80} & & \\
w. = \cancel{1440} & x = P & \\
\hline
\cancel{3} & \cancel{9} & \cancel{3} \\
\cancel{3} & \cancel{6} & \cancel{2} \\
\cancel{3} & \cancel{9} & \cancel{3} \\
\cancel{3} & \cancel{10} & \\
\cancel{3} & \cancel{12} & \cancel{4} \\
\cancel{3} & \cancel{18} &
\end{array}
$$

$x = 1$ ∴ One pound attached to the first wheel will keep 1440 attached to the last axle in equilibrio.

See Hutton's Mathematics, page 153.—Gregory's Math. page 214.

Example 9.

In a system of wheels and axles the radii of the wheels are 6, 9, 10, 12, 14, 16 inches, and those of the axles 3, 4, 5, 6, 7, 8; how many lb. applied to the first wheel, will balance one of the marble blocks intended for the triumphal arch at the new palace, weighing 23 tons.

Operation.

Their velocities are as 115 : 18.

$$
\begin{array}{r|ll}
23 & x & \\
\cancel{2}\ \cancel{20} & & \\
\hline
\cancel{3} & \cancel{6} & \\
\cancel{4} & 9 & \\
5 & \cancel{10} & \\
\cancel{6} & \cancel{12} & \cancel{3} \\
\cancel{7} & \cancel{14} & \cancel{2} \\
\cancel{8} & \cancel{16} & 2
\end{array}
$$

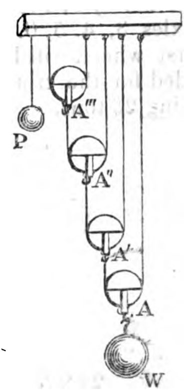
$$\therefore \frac{23 \times 5}{9 \times 2} = x = \frac{115}{18} = 6\tfrac{7}{18} \text{ cwt. Ans.}$$

OF THE PULLEY.

A pulley is a small wheel, commonly made of wood or brass, which turns about an iron axis passing through the centre, and fixed in a block, by means of a cord passed round its circumference, which serves to draw up weights. The pulley is either single or combined together, to increase the power. It is either fixed or moveable according as it is fixed to one place, or moves up and down with the weight and power.

If a power sustain a weight by means of a fixed pulley: the power and weight are equal.

When a power sustains a weight by a system of moveable pulleys, each embraced by a cord attached on one part to a fixed point, and on the other to the centre of the pulley next above it, as in the margin; then, if the cords are parallel to each other, each pulley gives a mechanical advantage of two to one, and the whole system an advantage denoted, by that power of two which is equal to the number of pulleys.



Example 10.

Suppose a system of six moveable pullies having fixed parallel cords attached to each, it is required to know how many pounds attached to the first will keep 1280 lb. in equilibrio, attached to the last.

$$\begin{array}{r|l} x & \cancel{1280} \\ & 20 \\ \hline 64 \left\{ \begin{matrix}2\\2\\2\\2\\2\\2\end{matrix} \right. & \begin{matrix}1\\ \\ \\ \\ \therefore x = 20.\ \text{Ans.}\\ \\ \end{matrix} \end{array}$$

See Gregory's Mathematics, page 215.

OF THE INCLINED PLANE.

The Inclined Plane is a plane inclined to the horizon, or making an angle with it. It is often reckoned one of the simple mechanic powers, and the double inclined plane makes the wedge. It is employed to advantage in raising heavy bodies in certain situations, diminishing their weights by laying them on inclined planes.

The power gained by the inclined plane is in proportion as the length of the plane is to its height. That is, when a weight is sustained on an inclined plane, by a power acting in the direction parallel to the plane, that the weight is in proportion to the power as the length of the plane is to its height.

Example 11.

If a man can draw two hundred pounds up a perpendicular wall 20 feet, how much could he draw along a plank 30 feet long, laid aslope?

$$\begin{array}{r|l} 30 & \\ \cancel{200} & \\ 10 & \\ \hline & x \\ x=30\times 10=300 & \cancel{20} \end{array}$$

OF THE SCREW.

The screw is a spiral thread or groove cut round a cylinder, and in every part making the same angle with its length; so that if the cylinder, with the spiral thread on it, were stretched, the spiral thread would be a straight line of an inclined plane, whose length would be to its height as the circumference of the cylinder is to the distance between the threads of the screws.

Example 12.

A lever 6 feet long is fixed at right angles to a screw, whose thread is 1 inch asunder, so that the lever turns once round in raising or depressing the screw 1 inch; if this lever be urged by a force of 50, with what force will the screw press?

The diameter of the circular orbit formed by the lever is 12 feet.

12 ft. = 144 in.

Then,

$$\begin{array}{r|l} 1 & 3{\cdot}1416 \\ \hline x & 144 \\ & 50 \end{array}$$

$x=3{\cdot}1416\times 144\times 50=22619{\cdot}52$ lb. Ans.

Example 13.

Suppose an endless screw is turned by a handle 30 inches long, the threads of the screw being half an inch asunder, turns a system of three toothed wheels and pinions, the diameter of the wheels is 24 inches, and that of the pinions 3 inches; what weight attached to the last pinion will a man raise working at the handle, supposing the man's power to be 200 lb.?

$$
\begin{array}{r|l}
\cancel{3} & \cancel{24}\ \ 8 \\
\cancel{3} & \cancel{24}\ \ \cancel{8} \\
\cancel{3} & \cancel{24}\ \ \cancel{8}\ \ \cancel{2} \\
\frac{1}{\cancel{2}} & 2 \\
\hline
\cancel{5}\ \ \cancel{20} & 3{\cdot}1416 \\
\cancel{112} & \cancel{60}\ \ 12 \\
7\ \ \cancel{14} & 200 \\
x &
\end{array}
$$

$$x = \frac{3{\cdot}1416 \times 8 \times 2 \times 12 \times 200}{7} = \frac{120637{\cdot}44}{7} =$$

17233·92 tons. See Hutton, vol. ii. p. 162.

TO FIND THE SPECIFIC GRAVITY OF BODIES.

Case 1.

When the body is heavier than water weigh it in water and in air, take their difference, and say, as the difference, or weight lost in water, is to its absolute weight, so is 1000, or the specific gravity of water, to the specific gravity of the body.

Examples for Illustration.

1. Required the specific gravity of a piece of cast iron, weighing 130 lb. in air, and 100 lb. in water; the specific gravity of water being 1000?

$$130-100=30$$

$$\begin{array}{c|c} 3\cancel{0} & 130 \\ \hline x & 100\cancel{0} \end{array}$$

$$x=\frac{13000}{3}=4333\tfrac{1}{3}.\ \text{Ans.}$$

2. What is the specific gravity of a piece of oak weighing in air 15 lb. but in water 10 lb.?

$$\begin{array}{c|c} \cancel{5} & \cancel{15} \\ & 3 \\ \hline x & 1000 \end{array}$$

$$\therefore x=1000\times3=3000.\ \text{Ans.}$$

Case 2.

When the body is lighter than water, attach a body to it heavier than water, so that their compound mass may sink. Weigh the denser body in water and in air and also the mass in water, and the lighter body in air; find how much the denser body lost in water, and also how much the two compounded lost, take their difference, and say, as the last difference is to the absolute weight of the lighter, so is 1000 to its specific gravity.

Examples for Illustration.

1. What is the specific gravity of fir, if to a piece weighing 20 lb. there be attached a piece of lead

weighing in air 30 lb., and in water 25 lb., and their compound weighing 20 lb. in water?

$$(30+20)=50-20=30$$
$$30-25=\ \ 5$$
$$\text{Diff. } 25$$

$\cancel{25}$	$\cancel{20}$ 4
$\cancel{5}$	
x	$\cancel{1000}$ 200

$\therefore x=200\times4=800.$ Ans.

2. What weight would balance a marble block so as to prepoise to move it, the dimensions being 63 feet long, 12 feet broad, and 12 feet thick; the specific gravity of marble being 2742?

Cubic foot 1	$\cancel{2742}$ oz. spec. grav.
oz. in ton. $\cancel{35840}$	1371
$\cancel{8960}$	
$\cancel{2240}$ x	$\cancel{63}$ 9
$\cancel{320}$	$\cancel{12}$ 3
160	$\cancel{12}$ 3

$$\therefore x=\frac{1371\times81}{160}=694\tfrac{11}{160}\text{ tons. Ans.}$$

3. What is the weight of one of the marble blocks which have been imported from Italy to build the triumphal arch which is to front the new palace; the dimensions being 22 feet long, 6 feet wide, and $2\frac{1}{2}$ feet

thick, and which required 23 horses to remove one from the wharf at Westminster; the specific gravity of marble being 2742 oz. avoirdupois.

	1	~~2742~~ oz.
oz. in ton.	~~35840~~	1371
	~~7168~~	
	~~3584~~	
	1792	~~22~~ 11
	x	~~6~~ 3
	~~2~~	~~5~~

$$\therefore x = \frac{1371 \times 11 \times 3}{1792} = 25\tfrac{443}{1792} \text{ tons. Ans.}$$

To find the specific gravity of a compound quantity mechanically united, by having the absolute weight of each of the parts and the corresponding specific gravity.

Let a, b, c, &c., be the weights of each mass.
— a', b', c', their specific gravities.
— w = the weight of water equivalent to the bulk a.

$$\text{Then } a' = \frac{a}{w} \text{ and } \frac{a}{a'} = w.$$

Now, the specific gravity of the whole mass being the sum of the weights of the separate masses in air divided by the sum of the weights of the equal bulks of water, we have.

$$S = \frac{a + b + c, \text{ \&c.}}{\dfrac{a + b + c}{a' + b' + c'} \text{ \&c.}}$$

That is, as the sum of the absolute weight in air, divided by the corresponding specific gravities, is to the sum of their absolute weight, so is unity to the specific gravity of the whole mass.

Example for Illustration.

What is the specific gravity of a mass composed of 6 lb. of steel, 4 lb. of copper, and 4 lb. of oak; their respective specific gravities being 7816,8876,950?

$$\begin{array}{r|l} \frac{96}{\cancel{7816}}+\frac{64}{\cancel{8876}}+\frac{64}{\cancel{950}} & 96+64+64=224 \\ \hline x & 1 \end{array}$$

when added,

$$\begin{array}{r|l} & 56 \\ \cancel{894480316} & \cancel{224} \\ 22362079 & 1029782425 \\ \hline x & 1 \end{array}$$

$$\therefore x=\frac{1029782425\times 56}{22362079}=2578\cdot 82 \text{ oz.}$$

PROMISCUOUS EXAMPLES.

Example 1.

A body weighing 20 lb. is impelled by such a force as would carry it 100 feet per second, with what velocity would a body of 8 lb. move if it were impelled by the same force?

$$\begin{array}{r|l} 5\ \cancel{20} & \\ \cancel{100} & \\ 50 & \\ \hline & x \\ & \cancel{8} \\ & \cancel{2} \end{array}$$

$x=50\times 5=250$ feet per second. Ans.

Example 2.

There are two bodies, of which one contains 25 times the matter of the other, but the less moves 1000 times the velocity of the greater; in what proportion are their momenta, or the forces with which they move?

$$\begin{array}{r|l} 1 & \\ \cancel{1000} & \\ 40 & \\ \hline & \cancel{25} \\ & 1\ x \end{array}$$

Ans. As 1 : 40. The less moves with a velocity 40 times that of the greater.

Example 3.

The battering-ram of Vespasian weighed 10000 lb.; and was found sufficient to demolish the walls of Jerusalem, being moved by the strength of hand, to pass through 20 feet in a second of time; with what velocity must a 32 lb. ball be discharged to do the same execution?

$$\begin{array}{r|l} 1250 & \\ \cancel{10000} & \\ \cancel{20} & \\ 5 & \\ \hline & x \\ & \cancel{32} \\ & \cancel{8} \end{array}$$

$\therefore x = 1250 \times 5 = 6250$ feet. Ans.

1

Example 4.

Suppose an iron bullet, weighing 42 lb., be discharged with a velocity of 3000 feet a second, make a breach in a garrison wall; with what velocity must a 28 pounder be discharged to do the same execution?

$$\begin{array}{r|l} \cancel{42}\ 3 & \\ \cancel{3000} & \\ 1500 & \\ \hline & x \\ & \cancel{28} \\ & \cancel{2} \end{array}$$

$x = 1500 \times 3 = 4500$ feet.

Example 5.

There are two bodies, the one of which weighs 100 lb. and the other 60 lb.; but the less body is impelled by a force 8 times greater than the other; required the proportion of the velocities with which they move?

$$\begin{array}{r|l} \cancel{60} & \cancel{100} \\ 3 & 5 \\ \hline x & 8 \\ & \\ & 3 : 40. \text{ Ans.} \end{array}$$

Example 6.

Suppose two labouring parties, one consisting of 40, and the other of 50 men, the strength of each man of the former to that of the latter is as 3 to 4; now if the first party could dig 100 cubic feet in 10 hours, in what time would the other party dig 480 feet, if the ground in the former case was twice as hard as in the latter? Ans. $14\frac{2}{5}$hs.

Example 7.

Having observed, from the Mansion House, Highgate, a piece of ordnance discharged on the river, and having heard the report 24 seconds after seeing the flash, required the distance I was from it? Ans. $5\frac{5}{22}$ miles.

Example 8.

Two men carrying 200 lb. of lead, attached to a pole between them, the ends of which rest on their shoulders, how much of this load is borne by each man, the weight hanging 6 inches from the centre, and the whole length of the pole being 4 feet?

$$\begin{array}{r|l}
\text{In.} & \\
\cancel{48} & \cancel{200} \\
\cancel{8} & 25 \\
\hline
x & 24+6=\cancel{30} \\
 & \qquad\quad 5
\end{array}$$

$\therefore x=25\times 5=125$, and $200-125=75$.

Hence 125 lb., and 75 lb. each will bear.

Proof.

$$\begin{array}{r|l}
5\ \cancel{30} & \\
\cancel{75} & \\
\cancel{25} & \\
\hline
 & \cancel{18}\ \ 3 \\
 & \cancel{125} \\
 & \cancel{25}
\end{array}$$

Example 9.

If a piece of stone weigh 14 lb., but in water weighs only $10\frac{1}{2}$ lb., required the specific gravity of the stone, that of water being 1000? Ans. 4000.

Example 10.

If 30 men in 40 days can dig 80 cubic yards, how many men who are stronger in proportion of 5 to 4 would be required to dig 120 yards in 90 days, supposing the ground in the latter case is harder than that in the former in the ratio of 9 to 8? Ans. 18.

Example 11.

Having 10 tons to raise with a handspike, and having formed a fulcrum 6 feet from it, how long must the handspike be that I may raise it with a power of 15 cwt.? Ans. 86 feet.

Example 12.

How much will the paving of a footpath cost, at 3s. 4d. the yard, the length being 45 feet, and the breadth 4 feet 8 inches?

$\cancel{9}$	$\cancel{40}$
3	$\cancel{45}$ 5
x	$4\frac{2}{3}=\cancel{14}$
$\cancel{240}$	7
3 $\cancel{6}$	

$$x=\frac{5\times7}{9}=\frac{35}{9}=\text{£}3\ 17\ 9\tfrac{1}{3}.$$

Example 13.

After observing a flash of lightning, it was 12 seconds before the thunder was heard; required the distance of the cloud from the observer?

$$\begin{array}{r|l} \text{sec. } 1 & \cancel{1100} \\ \cancel{5280} & 5 \\ \hline \cancel{44} & \cancel{12} \\ 2 \quad x & \end{array}$$

$$\therefore x = \tfrac{5}{2} = 2\tfrac{1}{2}. \text{ Ans.}$$

Example 14.

If a garrison of 5000 men be provided with provisions for 60 days at 14 oz. per day, how long would the provision last if 1000 men more were sent into the garrison, and the ratio were reduced to 12 oz. per man?

$$\begin{array}{r|l} \cancel{5000} & \\ 5 \quad \cancel{10} \quad \cancel{60} & \\ \cancel{14} & \\ 7 & \\ \hline & \cancel{6000} \\ & x \\ & \cancel{12} \quad 6 \quad 3 \end{array}$$

$$\therefore x = \frac{5 \times 5 \times 7}{3} = \frac{175}{3} = 58\tfrac{1}{3}. \text{ Ans.}$$

Example 15.

If two bodies hang freely on a pulley, the one 8 lb. and the other 6 lb., how far will the longer descend or the smaller ascend in 4 seconds after motion has commenced? Ans. $36\frac{16}{21}$ feet.

$$\text{Theorem } \frac{w - w}{w + w} \times 16\tfrac{1}{12} \times 16 = s$$

$$\therefore w + w : w - w :: 16\tfrac{1}{12} \times 16 : x.$$

Example 16.

Suppose I have 370 lb. to raise in a bucket, by a windlass whose radius is 10 inches; what power must I apply to a handle whose length is 36 inches?

Ans. $102\frac{7}{9}$ lb.

Example 17.

A garrison of 6000 men being furnished with provisions for 5 months, but, fearing an attack, 9000 men are sent to reinforce the place; how long can it hold out before the provisions are spent?

$\cancel{6000}$	
$2 \quad \cancel{5}$	
	$\cancel{3}$
	$\cancel{15000}$
	x

$\therefore x=2$ months. Ans.

Example 18.

What weight attached to a lever 3 feet from the fulcrum would balance 20 tubs of butter placed 9 feet at the other side a tub of butter weighing 58 lb.?

Ans. 3480 lb.

Example 19.

A bank of earth 330 yards long was to have been raised by 40 men in 7 days; but at the end of 5 days only 220 yards were completed; how many additional men must be employed to finish the bank in the proposed time?

men 10 $\cancel{40}$	$\cancel{2}$
days 5	$\cancel{220}$ yards
yards $\cancel{110}$	x men
	$\cancel{2}$ days.

$x=10\times5=50.$

$\therefore$ 10 additional men. Ans.

Example 20.

Sold a horse for 40 guineas, and thereby lost 4 per cent., whereas in dealing I ought to have gained 10 per cent.; how much was he sold for under his value?

Ans. $5\frac{5}{6}$ guineas.

Example 21.

Suppose a system of 9 wheels and axles act upon each other, the radii of the wheels being 10, 15, 20, 25, 30, 32, 36, 40, 45 inches, and the radii of the axles being each 6 inches; what power attached to the first wheel would keep in equilibrio 10 blocks of marble, each block 24 tons?

Ans. $1\frac{25119}{15625}$ lb.

Example 22.

Imported a quantity of dollars of plate from Cadiz, for which I have accepted bills at 38d. per dollar of exchange; now if 1200 dollars weigh 1000 ounces troy weight, and 64 dollars of plate equal 85 dollars of exchange, how much do they cost me per ounce, the charges being at $3\frac{3}{4}$ per cent.?

3 ~~6~~	~~5~~
~~1200~~	~~1000~~
17 ~~85~~	
~~415~~ ~~$103\frac{3}{4}$~~	
83 ~~83~~	4
19	
	~~64~~ ~~32~~ 16
	~~100~~ 20
oz. 1	1
	x

$$x=\frac{83\times19\times17\times3}{20\times16\times4}=\frac{80427}{1280}=62{\cdot}83 \text{ dol. Ans.}$$

Example 23.

A merchant at Lisbon takes a bill on Vienna for so many florins. At Lisbon the exchanges are 470 rees for one florin, 1000 rees for $49\frac{3}{4}$ pence sterling; he sends the bill to London, where it is negotiated at 10 fls. 3 kruitzers per pound sterling; required to know his gain or loss per cent. by sending the bill here instead of taking direct paper on London. Allow $\frac{3}{4}$ per cent.? Ans. Gained $1\frac{348249}{826657}$ per cent.

Example 24.

Purchased a bill on new York, which I have sent to my correspondent there, with orders to remit me the value in bills on London; the exchange between London and NewYork is 4s. 6d. per dollar with 10 per cent. in favor of New York. The charges on commission, interest, &c. being $2\frac{1}{2}$ per cent.; how much must the bill be sold for at New York that I may gain 1 per cent. by the transaction?

$$
\begin{array}{rr|ll}
\text{Cents} & \cancel{100} & \cancel{54}\text{d.} & 9 \\
 & 11\cancel{0} & \cancel{100} & \\
\cancel{207} & \cancel{103\frac{1}{2}} & \cancel{100} & 2 \\
\hline
23 & \cancel{240} & x & \\
 & \cancel{4} & & \\
 & 2 & &
\end{array}
$$

$x = 23 \times 11 \times 2 = 506$ $\therefore$ 506 cents per £ sterling.

Example 25.

If the average step of a horse be $2\frac{3}{4}$ feet, and that of a man $2\frac{1}{2}$ feet, how many men's paces are equal to 40 of a horse? Ans. 44.

Example 26.

If 840 men put in column of march, 7 men abreast, extend 360 paces, how many paces will they extend if they march 6 men abreast?

$$\begin{array}{r|l} & 60 \\ \cancel{840} & \cancel{360} \\ \cancel{7} & 7 \\ \hline x & \cancel{840} \\ \cancel{6} & \cancel{6} \end{array}$$

$x60\times7=420$ **paces. Ans.**

or,

$$\begin{array}{r|l} \cancel{120} & \cancel{3} \\ \cancel{840} & \cancel{360} \\ \cancel{7} & \cancel{7} \\ \hline \cancel{2}\quad x & \cancel{840}\quad 420 \\ \cancel{6} & 6 \end{array}$$

$x=420$ paces. **Ans.**

Example 27.

How many bricks, 9 inches long and 4 inches broad, will floor a room that is 20 feet square? **Ans. 1600.**

Example 28.

If 2 acres of land will maintain 3 horses for 4 days, how long will 5 acres maintain 6 horses? Ans. 5 days.

Example 29.

If 560 men are besieged in a garrison, and have provisions only for 3 months, how many men must leave the place, that the remainder may be supplied for 5 months? Ans. 224.

Example 30.

If a court-yard 33 square yards can be paved with 285 stones, that are each 15 inches by 10, how many stones, that are 24 inches by 18, will pave a court-yard that is 40 square yards? Ans. $119\frac{94}{99}$.

Example 31.

If a fathom of cable that is 5 inches circumference weigh 87 lb., how much would 10 fathoms weigh of the circumference of 10 inches?

1	87 lb.
$\cancel{5}$	
$\cancel{5}$	
x	$\cancel{10}$ 2
	$\cancel{10}$ 2
	10

$\therefore x = 87 \times 40 = 3480$ lb. Ans.

Example 32.

If a round pillar 7 inches in diameter be 4 feet, how much will be the diameter of a pillar of the same length containing 40 feet?

Solution.

7	4
7	
$\cancel{40}$	x^2
10	

$$\therefore x^2 = 7 \times 7 \times 10 = 490$$

$$x = \sqrt{490} = 22{\cdot}136 \text{ inches. Ans.}$$

Example 33.

A sack that holds 6 bushels when the diameter of the sack is 31½ inches, how much would a sack of the same length hold whose diameter is double that breadth?

Solution.

$$\begin{array}{c|c} \cancel{63} & 2 \\ \cancel{63} & 2 \\ & 6 \\ \hline x & \cancel{63} \\ & \cancel{63} \end{array}$$

$\therefore x = 6 \times 2 \times 2 = 24$ bushels. Ans.

Example 34.

If 3 gallons of brandy cost 19s. how much will 126 gallons cost?

Operation.

$$\begin{array}{r|l} \text{gls. } \cancel{3} & 19\text{s.} \\ \hline x & \cancel{126} \text{ gls.} \\ \cancel{20} & \cancel{42} \\ \cancel{10} & \cancel{21} \\ & 2{\cdot}1 \end{array}$$

$\therefore x = 19 \times 2{\cdot}1 = 39{\cdot}9 = £39\ 18.$ Ans.

The usual way.

$$\begin{array}{r} 3 : 19 :: 126 \\ 19 \\ \hline 1134 \\ 126 \\ \hline 3 \,|\, 2394 \\ \hline 2,0 \,|\, 79,8 \\ \hline £39 \,„\, 18 \end{array}$$

The converse.

35.

If 3 gallons of brandy cost 19s. how many gallons will £39 18 purchase?

Operation.

$$\begin{array}{r|l} 3 & \cancel{19}\text{ s.} \\ \hline \text{s. }\cancel{798} & x \\ 42 & \end{array}$$

$\therefore x = 42 \times 3 = 126.$ Ans.

The usual way.

$$\begin{array}{l} 19 : 3 :: 39_{\prime\prime}18 \\ 20 \\ \hline 798 \\ 3 \\ \hline 19 \mid 2394 \,(126 \\ 19 \\ \hline 49 \\ 38 \\ \hline 114 \\ 114 \\ \hline \end{array}$$

Example 36.

Sold cloth at £1 2 per yard, and gained 10 per cent. what would have been gained had it been sold at £1 5 per yard?

Operation.

$$\begin{array}{c|c} \cancel{22} & 25 \\ \hline x & \cancel{110} \\ & 5 \end{array}$$

$$\therefore x = 25 \times 5 = 125$$

Deduct 100

per cent. 25 Ans.

The usual way.

$$\begin{array}{ll} £ & s. \\ 1 & 2 \\ 20 & \\ \hline 22 & \end{array} : 110 :: \begin{array}{ll} £ & s. \\ 1 & 5 \\ 20 & \\ \hline 25 & \\ 110 & \end{array}$$

$$22\left\{\begin{array}{c} 2 \\ 11 \end{array}\right. \begin{array}{|l} 2750 \\ \hline 1375 \end{array}$$

125

Deduct, 100

25 Ans.

Example 31.

How much powder will fill a box whose length, breadth, and thickness, are 15, 12, and 10 inches?

Operation.

$$
\begin{array}{r|l}
\cancel{30} & 1 \text{ lb.} \\
\cancel{2} & \\
\hline
x & \cancel{15} \\
 & \cancel{12} \quad 6 \\
 & 10
\end{array}
$$

$\therefore x = 10 \times 6 = 60$ lb. Ans.

RULE.

As 30 inches are to 1 lb. so is the solid content of the cylinder, in inches, to the weight of powder.

This is an approximate rule founded on the supposition that 30 cubic inches of gunpowder, at a medium, weigh 1 lb. avóirdupoise.

Example 32.

How much powder will fill a cubical box whose side is 12 inches?

Operation.

$$
\begin{array}{r|l}
\cancel{30} & 1 \text{ lb.} \\
5 & \\
\hline
x & \cancel{12} \quad 2 \\
 & 12 \\
 & 12
\end{array}
$$

$$\therefore x = \frac{12 \times 12 \times 2}{5} = \frac{288}{5} = 57\tfrac{3}{5} \text{ lb. Ans.}$$

See HUTTON, vol. ii. p. 278.

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Example 33.

What length of a cylinder 8 inches in diameter, will be filled by 16 lb. of gunpowder ?

$$\begin{array}{cc|l} \cancel{4} & \cancel{8} & \cancel{16}\ \cancel{2} \\ & \cancel{8} & \\ \hline & x & \cancel{38{\cdot}2} \\ & & \end{array}$$

9·55 inches. Ans.

RULE.

As the square of the diameter is to the quantity of powder so are 38·2 to the length required.

See HUTTON, vol. ii., page 279.

This is an approximate rule founded on the supposition that 38·2 inches of gunpowder weigh 1 lb., different kinds of powder vary in their weights.

Example 34.

What length of a 36-pounder of $6\frac{2}{3}$ inches diameter will be filled by 12 lb. of powder ?

Operation.

$$\begin{array}{cc|l} \cancel{10} & \dfrac{\cancel{20}}{\cancel{3}} & 3 \\ \cancel{10} & \dfrac{\cancel{20}}{\cancel{3}} & 3 \\ \hline & & \cancel{12}\quad 6 \\ & x & \cancel{38}\ \cancel{{\cdot}2} \\ & & \cancel{19}\ \cancel{{\cdot}1} \\ & & {\cdot}191 \end{array}$$

$\therefore x = {\cdot}191 \times 6 \times 3 \times 3 = 10{\cdot}314$. Ans.

Example 35.

What length of a cylinder 8 inches diameter may be filled with 20 lb. of powder?

Operation.

$$
\begin{array}{r|l}
\cancel{8} & \\
\cancel{2}\ \ \cancel{8} & 1 \\
\hline
x & \cancel{20}\ \ 5 \\
 & \cancel{38}\ \ \cancel{\cdot 2} \\
 & \cancel{19}\ \ \cancel{\cdot 1} \\
 & 2{\cdot}3875
\end{array}
$$

$\therefore x = 2{\cdot}3875 \times 5 = 11{\cdot}9375$ inches. Ans.

See HUTTON, vol. ii. page 279.

Example 36.

Suppose two weights, 6 and 4 lb. connected with a chord which goes over a pulley, what space will the heavier descend, or the lighter ascend, in 4 seconds, friction not considered.

Operation.

$$
\begin{array}{r|l}
6+4=\cancel{10} & 6-4=\cancel{2} \\
5 & \\
\hline
3\ \ \cancel{12} & \cancel{16}\frac{1}{\cancel{12}}=\frac{193}{\cancel{12}} \\
x & \\
 & \cancel{4} \\
 & 4
\end{array}
$$

$$\therefore x = \frac{193 \times 4}{15} = 51\tfrac{7}{15}. \text{ Ans.}$$

Example 37.

If a person travel 300 miles in 10 days of 12 hours each, in how many days of 16 hours each may he travel 600 miles ?

Operation.

$$
\begin{array}{r|l}
5 \;\; \cancel{10} & \cancel{300} \\
3 \;\; \cancel{12} & \\
\hline
\cancel{600} & x \\
\cancel{2} & \cancel{16} \;\; \cancel{4} \;\; \cancel{2}
\end{array}
$$

$\therefore x = 5 \times 3 = 15$. Ans.

By the usual method.

First, if the days were of the same length, it would be, by direct proportion,

$$
\begin{array}{rr}
\text{As } 300 : 10 :: & 600 \\
 & 10 \\
\hline
3{,}00 \;| & 60{,}00 \\
\hline
 & 20 \text{ days.}
\end{array}
$$

But these would be days of 12 hours each, instead of 16, of which fewer will be required ; hence, by inverse proportion,

$$
\begin{array}{rrl}
12 : & 20 & :: \; 16 \\
 & 12 & \\
\hline
16 \;| & 240 & | \; 15 \text{ days.} \\
 & 16 & \\
\hline
 & 80 & \\
 & 80 & \\
\hline
\end{array}
$$

Example 38.

What is the interest of £560, for 5 years, at 4 per cent.?

Operation.

$$\begin{array}{cc|cc} \cancel{20} & \cancel{100} & & \\ \cancel{5} & 1 & \cancel{4} & \\ \hline & x & \cancel{560} & 112. \\ & & \cancel{5} & \end{array}$$

$\therefore x = £112.$ Ans.

The usual way.

$$\begin{array}{r} 100 : 4 :: 560 \\ 4 \\ \hline 100 \,|\, 2240 \\ \hline \end{array}$$

Interest for one year, 22·4

$$\begin{array}{lcccc} & \text{yr.} & £. & \text{s.} & \text{yr.} \\ \text{Then,} & 1 : & 22 & 8 :: & 5 \\ & & & 5 & \\ \hline \end{array}$$

Ans. £112

See GREGORY'S MATHEMATICS, page 43.

Example 39.

If 21 yards of cloth cost £24 10, how much will 160 cost?

Operation.

$$\begin{array}{r|ll} 3 & \cancel{24}\tfrac{1}{\cancel{2}} & \\ \cancel{21} & & \\ \cancel{2} & \cancel{49} & 7 \\ \hline x & \cancel{160} & \\ & 80 & \end{array}$$

$$\therefore x = \frac{80 \times 7}{3} = \frac{560}{3} = £186\ 13\quad 4.\ \text{Ans.}$$

The usual way.

$$\begin{array}{rrrl} \text{yds.} & £ & \text{s.} & \quad\text{yds.} \\ 21 : & 24 & 10 & :: 160 \\ & & & 4 \times 4 \times 10 = 160 \\ \hline & 98 & 0 & \\ & & 4 & \\ \hline & 392 & 0 & \\ & & 10 & \\ \hline \div 3\ | & 3920 & 0 & \\ \hline \div 7\ | & 1306 & 13 & 4 \\ \hline & £186 & 13 & 4\ \text{Ans.} \end{array}$$

See GREGORY, page 42.

Example 40.

How much would the gilding of the upright surface of the frustrum of a square pyramid come to, at 8d. the square foot, the slant height being 10 feet, each side of the greater end 40 inches, and each side of the less 26 inches?

Operation.

$$40+26=66\div 2=33 \text{ mean.}$$

$$\begin{array}{rr|ll}
\cancel{3} & \cancel{12} & & \\
 & \cancel{12} & \cancel{8}\text{d.} & \\
\hline
3 & \cancel{24} & \cancel{33} & 11 \\
 & \cancel{240} & \cancel{10} & \\
 & x & \cancel{12} & \\
 & & 4 & \\
\end{array}$$

$$\therefore x=\tfrac{11}{3}=\pounds 3 \quad 13 \quad 4. \text{ Ans.}$$

Example 41.

How much will the painting of a rhomboid come to, whose length is 37 feet, and height $5\frac{1}{4}$ feet, at 18d. the yard?

Operation.

$$\begin{array}{rr|ll}
 & \cancel{3} & \cancel{18} & \\
 & \cancel{3} & \cancel{3} & \\
\hline
40 & \cancel{240} & 37 & \\
 & x & \cancel{21} & 7 \\
 & 4 & & \\
\end{array}$$

$$\therefore x=\frac{37\times 7}{40\times 4}=\frac{259}{160}=\pounds 1 \quad 12 \quad 4\tfrac{1}{2}. \text{ Ans.}$$

Example 42.

If £$3\frac{1}{2}$ be the wages of 13 men for $7\frac{1}{2}$ days, what will be the wages of 20 men for $15\frac{1}{3}$ days?

Operation.

$$\begin{array}{r|l}
\cancel{2} & 7 \\
\text{Men } 13 & \\
3 \quad \cancel{15} & \cancel{2} \\
\hline
3 & \cancel{20} \quad 4 \\
x & 46
\end{array}$$

$$\therefore x = \frac{46 \times 28}{13 \times 9} = \frac{1288}{117} = £11\tfrac{1}{117}. \text{ Ans.}$$

Example 43.

Bought wool at 20d. per lb. and delivered it in barter at 28d. for broad cloth at 18s. per yard, at how much must I sell the cloth to gain 10 per cent. by the transaction?

Operation.

$$\begin{array}{r|l}
\cancel{20} & 8 \\
\hline
x & \cancel{100} \\
 & 5
\end{array}$$

$$\therefore x = 40. \text{ Ans.}$$

I gain 40 per cent. on the wool, consequently, by losing 30 per cent. I gain 10 per cent.

Then, $100 - 30 = 70$.

$$\begin{array}{r|l}
5 \quad \cancel{10} & 7 \\
\cancel{100} & \cancel{70} \\
\hline
x & \cancel{18} \\
 & 9
\end{array}$$

$$\therefore x = \tfrac{63}{5} = 12\text{s. } 7\tfrac{1}{5}\text{d. Ans.}$$

Example 44.

Bought broad cloth at 15s. per yard, and delivered it in barter at 20s., for tallow at 32s. per cwt. how much must the tallow be sold at per cwt. to clear 12 per cent. by the transaction ? Ans. 25s. $2\frac{2}{25}$d. per cwt.

Example 45.

Bought tea at 4s. per lb. and delivered it in barter at 5s. 6d. per lb., for wool at 3s. per lb., at how much must the wool be sold per lb. to gain 10 per cent. by the transaction ? Ans. 2s. $2\frac{1}{10}$d.

Example 46.

Two persons, A. and B. being on opposite sides of a wood, which is 536 yards about, they begin to go round it, both the same way, at the same instant of time; A. goes at the rate of 11 yards per minute, and B. 34 yards in 3 minutes; the question is, how many times will the wood be gone round before the quicker overtake the slower?

Operation.

$$\begin{array}{r|l} 1 & 3 \\ & 1 \text{ min.} \\ \hline x & \cancel{268} \\ \cancel{3} & \cancel{34} \\ \cancel{536} & 17 \\ 2 & \end{array}$$

$\therefore x = 17$ Ans.

Remark, B. goes at the rate of $11\frac{1}{3}$ yards per minute, and consequently gains $\frac{1}{3}$ yard per minute on A.

F 3

The usual way.

m.		yds.				yds.	
3	:	34	::	1	:	$11\frac{1}{3}$	B. walks per minute.

yds.

$11\frac{1}{3} - 11 = \frac{1}{3}$ B. gains per min. on A.

yds.		m.			
$\frac{1}{3}$	:	1	::	$\frac{536}{2}$	yards.

$\therefore \frac{1}{3} : 1 :: 268$

$$\begin{array}{r} 268 \\ 3 \\ \hline 804 \end{array}$$

804 minutes that B. must walk before he overtakes A.

	men.		yds.		men.
Then	3	:	34	::	804

$$\begin{array}{r} 804 \\ 34 \\ \hline 3216 \\ 2412 \\ \hline 3\,|\,27333 \\ \hline 9112 \end{array}$$

9112 yards that B. travels before he overtakes A.

	yds.		round.		yards.
As	536	:	1	::	9112

$$\begin{array}{r} 9112 \\ 1 \\ \hline 536\,|\,9112\,|\,17 \text{ rounds. Ans.} \\ 536\phantom{\,|\,17 \text{ rounds. Ans.}} \\ \hline 3752\phantom{\,|\,17 \text{ rounds. Ans.}} \\ 3752\phantom{\,|\,17 \text{ rounds. Ans.}} \end{array}$$

Example 47.

A wall was to be built 700 yards long, in 29 days; now, after 12 men had been employed on it for 11 days, it was found that they had completed only 220 yards of the wall, it is required to determine how many men must be added to the former, that the whole number of them may just finish the wall in the time proposed, at the same rate of working?

Ans. 4 to be added.

48. A. can do a piece of work alone in 12 days, and B. alone in 14; in what time will they both together perform a like quantity of work? Ans. $6\frac{6}{13}$ days.

49. A person who was possessed of a $\frac{3}{5}$ share of a copper mine, sold $\frac{3}{4}$ of his interest in it for £1800, what was the reputed value of the whole at the same rate? Ans. £4000.

50. How far off was the cloud from which thunder issued, whose report was 5 pulsations after the flash of lightning, counting 75 to a minute?* Ans. $1522\frac{2}{3}$ yds.

51. A common joist is 7 inches deep, and $2\frac{1}{2}$ broad, but I want a scantling just as big again, that shall be 3 inches thick; what will the other dimension be? Ans. $11\frac{2}{3}$ inches.

52. A ship's crew, consisting of 300 men, had provisions for 12 months, at the rate of 20 ounces per day; after having been at sea 6 months, they find that it will require 9 months more before they complete their voyage; how many ounces must each man have per day, that the provisions may last the 9 months, supposing that they had lost 50 men in an engagemen with pirates?

* Sound travels at the rate of 1142 feet per second. See Hutton, page 273, vol. ii.

Operation.

2 $\cancel{100}$	
men $\cancel{300}$	
mo. $\cancel{2}$ $\cancel{6}$	
oz. 4 $\cancel{20}$	
	$\cancel{5}$
	$\cancel{250}$ men.
	x oz.
	$\cancel{9}$ mo.
	$\cancel{3}$

$\therefore 4 \times 2 \times 2 = 16$ oz. Ans.

Example 53.

If 198 lb. of London be carried 8 miles for 25s. $7\frac{1}{5}$d. how much must be paid for conveying 480 lb. Amsterdam weight, 15 miles, if 100 lb. Amsterdam weight equal 110 lb. of London?

Operation.

$\cancel{11}$ $\cancel{86}$	32
lb. $\cancel{198}$	$\cancel{192}$
miles $\cancel{8}$	$\cancel{307\frac{1}{5}} = \frac{\cancel{1536}}{\cancel{5}}$ pence.
$\cancel{5}$	
x	$\cancel{480}$
5 $\cancel{240}$	$\cancel{110}$
$\cancel{100}$	$\cancel{15}$
	$\cancel{3}$

$\therefore x = \frac{32}{5} = £6$ 8s. Ans.

Example 54.

A wall 2000 feet long, $2\frac{1}{2}$ feet thick, is built with stones $\frac{1}{2}$ a foot long, $\frac{1}{4}$ of a foot broad, and $\frac{1}{8}$ of a foot thick, including mortar; the wall contained 4,000,000 stones, how high was the wall?

Operation.

$$
\begin{array}{r|l}
\cancel{2000} & \\
\cancel{5} & \cancel{2} \\
x & \\
\hline
 & 25 \\
 & \cancel{100} \\
 & \cancel{400} \\
\cancel{2} & \cancel{2000} \\
\cancel{4} & \cancel{4000000} \\
8 & 1 \\
\cancel{2} & 1 \\
 & 1
\end{array}
$$

Ans. $x = \frac{25}{2} = 12\frac{1}{2}$ feet.

Example 55.

What is the weight of a block of dry oak, which is 10 feet long, 3 feet broad, and $2\frac{1}{2}$ feet deep or thick; the specific gravity of oak being 925?

Ans. $4335\frac{15}{16}$ lb.

56. An irregular piece of lead ore weighs in air 12 ounces, but in water only 7; and another fragment weighs in air $14\frac{1}{2}$ ounces, but in water only 9; required their comparative densities, or specific gravities?

Ans. As 145 to 132.

57. An irregular fragment of glass, in the scale, weighs 171 grains, and another of magnet 102 grains; but, in water, the first fetches up no more than 120 grains, and the other 79; what will be the ratio of their specific gravities?

Ans. glass to magnet as 437 to 578, or nearly as 10 to 13.

58. Suppose, that a man-of-war with its ordnance, rigging, and appointments, sinks so deep as to displace 50,000 cubic feet of fresh water, what is the whole weight of the vessel? Ans. $1395\frac{1}{10}$ tons.

59. Suppose two weights, one of 6, the other of 2 pounds, to be suspended upon a pin, by means of a string, to determine how far the greater will descend, and the other ascend, in 1 second of time, neglecting the friction, &c.? Ans. $8\frac{1}{24}$ feet.

60. What weight can a man raise with a handspike, or iron crow, 8 feet long, if the fulcrum, or prop, be 5 inches from one end, and he presses with a force equal to 150 lb. at the other? Ans. 2730 lb.

61. If one arm of a steel-yard be 3 inches, what must be the length of the other, that a counterpoise of 10 lb. may be sufficient to weigh a 1 cwt. supposing the weight of the instrument itself is not considered.

Ans. 33·6 inches.

62. The cylinder, or axle, over a common draw-well, is 3 inches in diameter, the rope $\frac{3}{4}$ of an inch in diameter, and the handle describes a circle of 30 inches in diameter; now, what weight can a man draw up, who acts with a force equal to 40 lb.?* Ans. 320 lb.

63. If the screw of a press be turned with a lever 7 feet long, and the threads of the screw are 1 inch asunder, what is the force of the press when the power at the end of the lever is 100 lb. supposing the screw to act without friction? Ans. 52779 lb. nearly.

* As power (40) : weight (x) :: radius cylinder ($1\frac{1}{2}$) + radius of the rope $\frac{3}{8}$: radius of the handle (15.)

64. To determine the weight of a tapering beam of timber, 30 feet long, we found that it rested in an horizontal position, on a prop, or fulcrum, 16 feet from the less end, but when the middle of the beam was brought over the prop, it required the weight of a man, which was 200 lb. at the less end, to keep it in equilibrio, hence the weight is required. Ans. 3000 lb.

65. The declination of the moon, on the 5th of September, at noon, was 3° 39′ 19″, and at midnight 1° 19′ 34″, what was her declination at 10 o'clock?

Ans. 1° 56′ 27·5″.

66. On the 5th of September at noon the sun's declination was 7° 1′ 35″, and on the 6th it was 6° 39′ 19″, what was the true declination at the 20th hour?

Ans. 6° 43′ $1\frac{2}{3}$′.

67. The moon was within 16 degrees of a bright star, at 8 o'clock in the evening; now, if she moves at the rate of 13 degrees in her orbit, per day, towards the star, how near will she be to it at 2 o'clock in the morning? Ans. 12° 45′.

68. Three merchants joined in partnership, A. put in £6000, B. £4000, C. £5000; they gained by trade £500, how much is each to receive?

Ans. A. £200, B. £133 6 8, C. £166 13 4.

69. How much would the paving of a rectangular court yard come to, at 2s. 6d. the square yard, the dimensions being 120 feet long, and 88 feet wide?

Ans. £146 13 4.

70. How much must be put at interest for 7 years, to gain £500, at 5 per cent. per annum.

Ans. £1428 11 $5\frac{1}{7}$.

71. If I buy goods at 18s. and sell them at 24s. how much do I gain per cent. per annum? Ans. $33\frac{1}{3}$.

72. How many yards of crimson drugget, $\frac{9}{4}$ wide, will be required to cover an elliptical room, whose axes are 28 and 18 yards? Ans. 175·9296 yds.

73. How much would the carpetting of a circular room come to, the diameter being 24 feet, and the carpetting $\frac{7}{4}$ wide, at 5s. the yard? Ans. £7 3 7·392.

74. How many solid feet of wood are in 150 planks, the length of each being 30 feet, the breadth 3 feet, and the depth $2\frac{1}{2}$ feet? Ans. 33750 solid feet.

75. Four merchants joined in an adventure to Mexico, A. put in £1200, B. £1500, C. £500, D. £1000, they gain £2400, how much is each man's gain?
Ans. A. £685 14 $3\frac{3}{7}$, B. £857 2 $10\frac{2}{7}$,
C. 285 14 $3\frac{3}{7}$, D. £571 8 $6\frac{6}{7}$.

76. How many stones, 15 inches long, and 6 inches broad, would pave a hall 10 yards long, and 6 yards wide? Ans. 864 stones.

77. How many bricks, 10 inches long, 4 inches broad, and 3 inches thick, will build a wall 64 yards long, 3 feet thick, and 8 feet high?
Ans. $66355\frac{1}{5}$ bricks.

78. How many shares in the Imperial Gas Association can I purchase for £15000, in the 3 per cent. consols, market price at $78\frac{5}{8}$, the shares being at £15 premium (100 shares) all the instalments paid up?
Ans. 102·228 shares.

79. Suppose I have 12 lb. and 8 lb. attached to a string, which passes over a fixed pulley, how far will the one descend, or the other ascend, in 2 seconds?
Ans. $12\frac{13}{15}$.

80. Bought tea at 4s. per lb. which I deliver in barter at 6s. for Irish linen at 4s. 6d. per yard; at how much must I sell the linen per yard, to gain 10 per cent. by the transaction? Ans. 2s. $8\frac{2}{5}$d. per yd.

81. Bought Madeira at £42 a pipe, which I barter at £50 for scarlet cloth at 18s. per yard, at how much must I sell it per yard to gain 15 per cent. by the transaction? Ans. 17s. $3\frac{1}{4}$d

ANNUITIES.

TABLE I.

Shewing the present value of an Annuity of One Pound for any Number of Years, not exceeding 100, *at* 5 *and* 6 *per Cent.*

yrs	5 per Ct.	6 per Ct.	yrs	5 per Ct.	6 per Ct.	yrs	5 per Ct.	6 per Ct.
1	·9524	·9434	35	16·3742	14·4982	69	19·3098	16·3676
2	1·8594	1·8334	36	16·5469	14·6210	70	19·3427	16·3845
3	2·7232	2·6730	37	16·7113	14·7368	71	19·3740	16·4005
4	3·5460	3·4651	38	16·8679	14·8460	72	19·4038	16·4156
5	4·3295	4·2124	39	17·0170	14·9491	73	19·4322	16·4298
6	5·0757	4·9173	40	17·1591	15·0463	74	19·4592	16·4432
7	5·7864	5·5824	41	17·2944	15·1380	75	19·4850	16·4558
8	6·4632	6·2098	42	17·4232	15·2245	76	19·5095	16·4678
9	7·1078	6·8017	43	17·5459	15·3062	77	19·5329	16·4790
10	7·7217	7·3601	44	17·6628	15·3832	78	19·5551	16·4897
11	8·3064	7·8869	45	17·7741	15·4558	79	19·5765	16·4997
12	8·8633	8·3838	46	17·8801	15·5244	80	19·5965	16·5091
13	9·3936	8·8527	47	17·9810	15·5890	81	19·6157	16·5180
14	9·8986	9·2950	48	18·0772	15·6500	82	19·6340	16·5265
15	10·3797	9·7122	49	18·1687	15·7076	83	19·6514	16·5344
16	10·8378	10·1059	50	18·2559	15·7619	84	19·6680	16·5419
17	11·2741	10·4773	51	18·3390	15·8131	85	19·6838	16·5489
18	11·6896	10·8276	52	18·4181	15·8614	86	19·6989	16·5556
19	12·0853	11·1581	53	18·4934	15·9070	87	19·7132	16·5619
20	12·4622	11·4699	54	18·5651	15·9500	88	19·7269	16·5678
21	12·8212	11·7641	55	18·6335	15·9905	89	19·7399	16·5734
22	13·1630	12·0416	56	18·6985	16·0288	90	19·7523	16·5787
23	13·4886	12·3034	57	18·7605	16·0649	91	19·7641	16·5837
24	13·7986	12·5504	58	18·8195	16·0990	92	19·7753	16·5884
25	14·0939	12·7834	59	18·8758	16·1311	93	19·7860	16·5928
26	14·3752	13·0032	60	18·9293	16·1614	94	19·7962	16·5970
27	14·6430	13·2105	61	18·9803	16·1900	95	19·8059	16·6009
28	14·8981	13·4062	62	19·0288	16·2170	96	19·8151	16·6047
29	15·1411	13·5907	63	19·0751	16·2425	97	19·8239	16·6082
30	15·3725	13·7648	64	19·1191	16·2665	98	19·8323	16·6115
31	15·5928	13·9291	65	19·1611	16·2891	99	19·8403	16·6146
32	15·8027	14·0840	66	19·2010	16·3105	100	19·8479	16·6175
33	16·0025	14·2302	67	19·2391	16·3307			
34	16·0929	14·3681	68	19·2753	16·3497			

ANNUITIES COMPUTED AT COMPOUND INTEREST.

CASE I.

Annuity, Time, and *Rate*, being given to find the present value.

RULE.

Find in Table I. the present value of £1 per annum for the given time, at the given rate. Multiply it by the annuity for the value required.*

Example for Illustration.

What is the present value of £100 annuity, to continue 20 years, interest being at 5 per cent. per annum?

The present value of £1 annuity for 20 years, at 5 per cent. per annum = 12·4622
Multiply by 100

£1246·22

Or,

$$\frac{1}{x} \Big| \frac{\cancel{12{\cdot}4622}\ 1246{\cdot}22}{\cancel{100}}$$

x = £1246·22. Ans.

Examples for Practice.

1. How much is the present value of £30 . 4, to continue 7 years, interest at 5 per cent.?
Ans. £174 14 11·8272.

2. How much will be the purchase of an annuity of £974 7 4, to continue 40 years, interest being at 6 per cent.? Ans. £14660 10 3·088.

3. How much is the present worth of £9320 per annum, to continue 30 years, interest at 6 per cent.?
Ans. £128287 18 8·64.

* See Scholium I. page 197, Improved System of Arithmetic,

CASE II.

Principal, or *purchase-money*, *rate*, and *time*, being given, to find the annuity.

RULE.

Divide the purchase money by the present value of £1, at the given rate for the given time, for the annuity required.

Example for Illustration.

What annuity, to continue 20 years, can be purchased for £1146 19 10, interest at 6 per cent.?

The present value of £1 annuity, to continue 20 years, at 6 per cent. = £11·4699.

Therefore,

£1146 19 10 = 1146·99 ÷ 11·4699 = £100. Ans.

Examples for Practice.

1. A farmer having had his lease renewed for 19 years, by paying £200 fine, but the landlord agreeing to have an additional equivalent rent laid on; how much will it be, interest at 5 per cent.?

Ans. £16 10 11·76.

2. A farmer takes a lease of a farm for 31 years, at a yearly rent of £400; he lays out £1500 in draining, hedging, manuring, &c.: at what annual rent must he let it, to be repaid his principal, with compound interest at 5 per cent. within the time?

Ans. £496 3 11½.

3. Purchased the lease of a mansion for £600, at an annual rent of £300; laid out in painting and repairs £600; how much additional rent must I lay on it, to be repaid the £1200, with compound interest, at 5 per cent. per annum, in 21 years? Ans. £93 11 10¾.

CASE III.

Fine, yearly rent, rate of interest, and *time,* being given, to find the *fine* that must be paid to reduce the rent to any proposed sum.

RULE.

The present value of the difference between the actual rent and that to which it is to be reduced, will be the fine required.

Example for Illustration.

How much must be paid for a lease of 30 years, at a yearly rent of £46, to reduce the rent to £30, interest at 6 per cent.?

46
30
——
Difference, 16

The annuity for 30 years, at 6 per cent. = 13·7648
Multiply by 16

825888
137648

220·2368
20

4·7360
12

Ans. £220 4 8·832. 8·832

Examples for Practice.

1. If £500 be paid for a lease of 61 years, at a rent of £120, how much should be paid to reduce the rent to £70, interest at 5 per cent.?

Ans. £1449 0 3·6.

2. If I take the lease of a house for 41 years, at an annual rent of £90, and lay out £700 in repairs; now if that sum be left to diminish the rent, how much will be the yearly rent, interest at 5 per cent.?

Ans. £49·5245.

3. If the purchase of the lease of a farm for 31 years, at the yearly rent of £400, be £500; how much would the rent be if I had paid £2500, interest at 5 per cent.? Ans. £271·7357.

CASE IV.

To find the value of annuities in reversion. *Annuity, time in reversion*, and *rate*, being given, to find the *present value*.

RULE.

Find the value of £1 annuity (Table I.) at the given rate, both for the time in reversion and the time in being; take their difference, and multiply it by the annuity for the value required.

Example for Illustration.

How much is the present value of £50, for 14 years, which are to commence after 7 years, interest at 5 per cent.?

The value of £1 annuity for 21 years =	12·8212
The value of £1 annuity for 7 years =	5·7864
Difference -	7·0348
Multiply by -	50
	351·74
	20
	14·8
	12
	9·6

Ans. £351 14 9·6.

Example for Practice.

A. has a term of 7 years in an estate worth £50 per annum; B. has a term of 14 years in the same estate, after the 7 years are expired; C. has a further term of 10 years, after the expiration of the 21 years; how much is the present value of the several terms, interest at 5 per cent.? Ans. A. £289·32; B. £351·74; C. £138·58.

To find the value of Freehold Estates, to be bought or sold for ever.

RULE.

Multiply the yearly rent by 100, and divide by the rate, for the value required.

Example for Illustration.

How much is an estate, producing £252 per annum, worth, at 6 per cent. per annum?

$$\frac{252 \times 100}{6} = £4200. \text{ Ans.*}$$

* Find the fee simple for £1, and multiply it by the annuity; thus, $\frac{110}{6}$ = £16 13 4, which, being multiplied by 252 = £4200.

TABLE II.

Shewing the value of an Annuity on a Single Life at every Age, according to the Carlisle Table of Mortality.

Age.	5 per ct.	6 per ct.	Age.	5 per ct.	6 per ct.	Age.	5 per ct.	6 per ct.
1	13·995	12·078	35	14·127	12·573	69	6·643	6·277
2	14·983	12·925	36	13·987	12·465	70	6·336	5·998
3	15·824	13·652	37	13·843	12·354	71	6·015	5·704
4	16·271	14·042	38	13·695	12·239	72	5·711	5·424
5	16·590	14·325	39	13·542	12·120	73	5·435	5·170
6	16·735	14·460	40	13·390	12·002	74	5·190	4·944
7	16·790	14·518	41	13·245	11·890	75	4·989	4·760
8	16·786	14·526	42	13·101	11·779	76	4·797	4·579
9	16·742	14·500	43	12·957	11·668	77	4·609	4·410
10	16·669	14·448	44	12·806	11·551	78	4·422	4·238
11	16 581	14·384	45	12·648	11·428	79	4 210	4·040
12	16·494	14·321	46	12·480	11·296	80	4·015	3·858
13	16·406	14·257	47	12·301	11·154	81	3·799	3·656
14	16·316	14·191	48	12·107	10·998	82	3·606	3·474
15	16·227	14·126	49	11·892	10·828	83	3·406	3·286
16	16·144	14·067	50	11·660	10·631	84	3·211	3·102
17	16·066	14·012	51	11·410	10·422	85	3·009	2·909
18	15·987	13·956	52	11·154	10·208	86	2·830	2·739
19	15·904	13·897	53	10·892	9·988	87	2·685	2·599
20	15·817	13·835	54	10·624	9·761	88	2·597	2·515
21	15·726	13·769	55	10·347	9·524	89	2·495	2·417
22	15 628	13·697	56	10·063	9·280	90	2·339	2·266
23	15·525	13·621	57	9·771	9·027	91	2·321	2·248
24	15·417	13·541	58	9·478	8·772	92	2·412	2·337
25	15·303	13·456	59	9·199	8 529	93	2·518	2·440
26	15·187	13·368	60	8·940	8·304	94	2·569	2·492
27	15 065	13·275	61	8·712	8·108	95	2·596	2·522
28	14·942	13·182	62	8·487	7·913	96	2·555	2·486
29	14·827	13·096	63	8·258	7·714	97	2·428	2·368
30	14·723	13·020	64	8·016	7·502	98	2·278	2·227
31	14·617	12·942	65	7·765	7·281	99	2·045	2·004
32	14·506	12·860	66	7·503	7·049	100	1·624	1·596
33	14·387	12·771	67	7·227	6·803			
34	14·260	12·675	68	6·941	6·546			

The valuation of Annuities on one Life.

RULE.

Multiply the year's purchase of an annuity on a single life, (Table II.) by the annuity for the value required.

Example for Illustration.

How much is the value of an annuity of £50 upon a life of 50 years, at 5 per cent.?

50 years = 11·66 years' purchase.

Therefore $11{\cdot}66 \times £50 = £583$. Ans.

Examples for Practice.

1. How much is the value of £700 per annum, on a life of 27 years, at 5 per cent.? Ans. £10545 10.

2. A gentleman marries a lady 21 years of age, whose fortune is £1500 per annum, during her life; and being desirous of converting it into ready-money, how much is he to receive, interest at 5 per cent.? Ans. £23589.

PROBLEM I.

To find the value of an Annuity granted on the joint continuance of two unequal lives.

Let P = the years' purchase of one life. (Table II.)
S = that of the other.
r = the amount of £1 for 1 year, at the given rate.

Then,

Theorem 1. $\dfrac{SP}{(S+P)-(r-1\times SP)}$ = years' purchase on a joint continuance.

Example for Illustration.

How many years' purchase is an annuity worth, granted on the joint continuance of the two lives, 40 and 50. Ans. 9·0542 years' purchase.

Example for Practice.

How much is the present value of an annuity of £500, on the joint continuance of a gentleman, aged 35, and his wife, aged 27, at 5 per cent.?

Ans. £5736 4.

PROBLEM II.

To find the value of two equal lives.

Theorem 2. $\frac{P}{2-(r-1)\times P}$ = the year's purchase.

Example for Practice.

How much is the value of an annuity of £300, on the joint continuance of two lives, of 25 years each, at 5 per cent.? Ans. £3717 15.

PROBLEM III.

Two single lives being given, to find the value of an annuity that is to continue as long as either of them is in being.

Let P and S represent the two lives.

Theorem 3. $P+S-(PS)$ in which (PS) represents the value of their joint lives.

Example for Illustration.

How much is the value of the longest of two lives, aged 10 and 30, at 5 per cent?

The purchase of 10 years, Tab. II. = 16·669
Ditto 30 years, ——— = 14·723

31·392 = $P+S$
The purchase of 2 joint lives, Theo. 1. = 12·8349 = (PS)

18·5571

Ans. 18·5571 years' purchase.

Examples for Practice.

1. What is the value of the longest of two lives, aged 30 and 40. Ans. 17·3144 years' purchase.

2. What annuity can I purchase for 14 years, for £15,000, in the 3 per cent. consols, market price $87\frac{5}{8}$, the value of money being at 5 per cent. per annum? Ans. £1324 1 0·216.

3. A lady 47 years of age, having £10,000 in the 5 per cent. navy annuities, market price $105\frac{1}{4}$, wishes to purchase an annuity during the remainder of her life; how much is the annuity to be, the interest of money being at 5 per cent. per annum? Ans. £853 11 7·2.

4. A gentleman 37 years of age, having an annuity of £800, during the remainder of his life, wishes to dispose of it, and to invest the value of it in the Eagle Insurance Association, the shares being at £40 premium, interest of 5 per cent.? Ans. £819·6.

EXCHANGE.

When an equivalent in the money of one country is paid or received in the money of another, the transaction is called Exchange.

The course of Exchange, which is distinguished into *certain* and *uncertain*, is always calculated in the uncertain price. Thus, London gives One Pound, a fixed though imaginary sum, for the fluctuating value of the monies of France, Holland, Hamburgh, &c.; and for this reason the course of Exchange is reckoned in the uncertain price of the fluctuating value. But if London give a variable sum for the piastre of Spain, the pezza of Leghorn, or the milree of Portugal; the course of Exchange is computed in the money of England.

The fluctuation in the course of Exchange is chiefly occasioned by the fluctuation of the balance of trade; that is, by the difference between the commercial exports and imports of any two places or countries.

The usual method of transacting Exchanges is by a bill, or written order, for the payment of some specified amount at a time appointed. The persons primarily concerned in a bill of Exchange are, 1st. The *Drawer*, who gives the order; 2d. The *Drawee*, to whom the order is addressed, and who, when he engages to pay the bill, is termed the *Acceptor*; 3d. The *Payee*, to whom the specified amount is directed to be paid.

The persons subsequently having an interest in the bill, and who may be one or other of the former, are the Negociator, Remitter, Holder or Possessor, &c.

The term of a bill varies according to the special agreement, or the custom of the respective countries. Although some bills are drawn payable at sight, and others at some fixed number of days, yet, in general, the term mentioned in the bill is *Usance*, or *Half-Usance*; and sometimes *Double Usance*.

Usance means the customary term of bills between the two places concerned, and is either one, two, or three calendar months after sight, or after date. On all bills drawn at date, but not on those at sight, days of grace are allowed, which vary in different countries; three are allowed in the British dominions and in America.

The *Par* of Exchange is the comparative intrinsic value, with respect to weight and fineness, of the monies in circulation in different countries.

AMSTERDAM.

In 1816 a new monetary system was introduced in the kingdom of the Netherlands, consisting of florins and cents. $2\frac{1}{2}$ florins$=$1 rixdollar.

Usance is generally one month after date; and days of grace, which are nominally six, are seldom taken.

Example 1.

Reduce 40,000 florins 5 cents. into British money, at 10 florins 50 cents. per £ sterling.

$$\begin{array}{c|c} \cancel{10{,}50} & 1 \\ 2\cdot1 & \\ \hline x & \cancel{40000\cdot05} \\ & 8000\cdot01 \end{array}$$

$$\therefore x = \frac{8000\cdot01}{2\cdot1} = £3809\ \ 10\ \ 6\cdot8568. \text{ Ans.}$$

2. How many florins and cents. are equivalent to £745 15, at 10 fr. 25 cents per £.? Ans. 7643·94.

HAMBURGH.

At Hamburgh there are two sorts of money, the *Banco* and *Currency*, with agio from 18 to 25 per cent., but of which the par is about 23. Accounts are kept in marks and shillings of both values, and also in pounds, shillings, and pence, *Flemish*.

Usance for bills drawn from England, France, and Holland, is one month after date; those drawn from Spain, Portugal, and Italy, two months, with twelve days' grace.

TABLE OF HAMBURGH MONEY.

Flemish.

6 pennings	=	1 groot, or penny.
12 pence	=	1 shilling.
20 shillings	=	1 pound.

Hambro'.

12 pennings	=	1 sol, or shilling.
16 shillings	=	1 mark.
2 marks	=	1 dollar of Exchange.
3 marks	=	1 rixdollar.

Hence 6 shillings *Hambro'* = 1 shilling *Flemish*,
And 1 mark banco = 32 pence *Flemish*;
3 marks are equivalent to 8 shillings *Flemish*.

Example for Illustration.

Reduce 200 marks current into pounds sterling, exchange at 35s. 6d. Flemish Banco, and *agio* 20 per cent.

3	50
cur. ~~120~~	~~100~~ banco.
banco 1	32d. Flem.
Fl. ~~426~~	£1 ster.
213	
x	~~200~~
	5

$$x = \frac{50 \times 32 \times 5}{213 \times 3} = £12{\cdot}51953 \text{ ster. Ans.}$$

Example for Practice.

1. Reduce £900 sterling to marks and shillings banco, exchange at 35¼s. Flemish per £ sterling.

Ans. 11896 marks 14 sh.

PARIS.

Accounts in France are now kept in francs and centimes, but were formerly kept in *livres*, *sols*, and *deniers*.

Usance is 30 days, not including the date. No days' grace are allowed throughout all France; but a *delais* is allowed under certain circumstances.

TABLE OF FRENCH MONEY.

10 centimes	=	1 decime.
10 decimes	=	1 franc.

Note.—80 *francs are equivalent to* 81 *livres Tournais.*

Example for Illustration.

Reduce 1800 francs 45 centimes to English money, exchange at 24 francs per £ sterling.

fr. ~~24~~	
~~3~~	£
x	~~1800·45~~ fr.
	~~225·05625~~
	75·01875

$\therefore x = £75{\cdot}01875$. Ans.

Example for Practice.

1. Reduce £1457 10, to francs and centimes, exchange at 23½ francs per £ sterling.

Ans. 34251 fr. 25 cents.

MADRID.

At Madrid, and other places in Spain, accounts are chiefly kept in piastres, reals, and maravedis.

Exchanges between England and Spain are for the most part negociated by the piastre or dollar of exchange, for which London gives a variable number of pence sterling.

Usance for bills drawn from England, France, Genoa, and Leghorn, is 60 days after date; from Amsterdam and Hamburgh two calendar months. The days of grace are fourteen, provided the bills are accepted before they become due, otherwise they must be paid or protested immediately.

TABLE OF SPANISH MONEY.

34 maravedis	=	1 real.
8 reals	=	1 piastre or dollar of exchange.
4 piastres	=	1 pistole of exchange.
375 maravedis	=	1 ducat of exchange.

Example for Illustration.

Reduce 1090 piastres 7 reals to sterling money, exchange at 36d. per piastre.

$$\begin{array}{r|l} \text{reals } 8 & \cancel{36} \\ \cancel{240} & \cancel{9}\quad 3 \\ \cancel{20}\quad \cancel{60} & \\ \hline x & \cancel{8727} \\ & \cancel{1090\cdot875} \\ & 54\cdot54375 \end{array}$$

$\therefore x = 54\cdot54375 \times 3 = 163\cdot63125 = £163\ 12\ 7\frac{1}{2}$. Ans.

Example for Practice.

In 19528 reals how many pounds sterling, exchange at $41\frac{7}{8}$ pence per piastre? Ans. £425 18 $0\frac{7}{8}$.

LISBON.

In Portugal accounts are kept in milrees and rees.

The usance allowed on bills drawn from Spain is fifteen days' sight, from London thirty days' sight; from Germany and Holland two months after date; from France 60 days after date; from Italy and Ireland 3 months after date.

Six days' grace are allowed on all foreign bills when they have been accepted. But unaccepted bills must be paid or protested on the day they expire.

TABLE OF PORTUGUESE MONEY.

1000 rees	=	1 milree.
400 rees	=	1 crusado.
480 rees	=	1 new crusado.

Example for Illustration.

Reduce 280 milrees 800 rees to English money, exchange at 65d. per milree.

$$
\begin{array}{r|l}
1 & 65 \\
\cancel{3}\ \ \cancel{240} & \\
\hline
x & \cancel{280{\cdot}8} \\
 & \cancel{3{\cdot}51} \\
 & 1{\cdot}17
\end{array}
$$

$x = 1{\cdot}17 \times 65 = £76\ \ 1.$ Ans.

Example for Practice.

What sum, in British money, is equivalent to 1463 milrees 250 rees, exchange at 60d. per milree?

Ans. £365 16 3.

LEGHORN.

At Leghorn accounts are kept, principally by Merchants and Bankers, in piastres, soldi, and denari.

Usance is three months after date, with fourteen days' grace.

TABLE.

12 denari = 1 soldo.
20 soldi = 1 piastre, pezza, or dollar.

Example for Illustration.

Reduce 630 piastres 12 soldi to English currency, exchange at 50d. per piastre.

$$
\begin{array}{r|l}
\text{pias. } 1 & 5\cancel{0} \\
\cancel{240} & \\
\cancel{4} & \\
\hline
x & \cancel{630{\cdot}6} \\
 & \cancel{105{\cdot}1} \\
 & 26{\cdot}275
\end{array}
$$

$x = 26{\cdot}275 \times 5 = 131.375 = £131\ \ 7\ \ 6.$ Ans.

Example for Practice.

Reduce £131 7 6 to the money of Leghorn, exchange at 50d. per piastre or pezza.

Ans. 630 pi. 12 sol.

VENICE.

There were formerly three kinds of money in use in Venice; viz. *moneta piccola*, *valuta corrente*, and *banco*. Moneta piccola has been in use since 1750; valuta corrente preceded it; and banco was the money in which the Bank kept accounts.

Usance, for bills drawn from Hamburgh, Holland, France, and Spain, is two months after date; from Lisbon and London three months after date; from Ancona and Rome ten days after acceptance; from Milan twenty days after date; from Genoa, Naples, Sicily, and Vienna fifteen days after acceptance; from Florence and Leghorn five days after acceptance.

MONETA PICCOLA.

12 denari = 1 soldo.
20 soldi = 1 lira.

VALUTA CORRENTE.

12 grossetti = 1 grosso.
24 grossi = 1 ducato.

The ducat banco is also divided into 24 grossi, each grosso into 12 denari: $6\frac{1}{5}$ lire piccole equal the ducat corrente, and $9\frac{3}{5}$ lire the ducat banco.

The new method of keeping accounts is in lire Italiane, which is divided according to the French system into 100 centimes; 405 lire piccole are worth 207 lire Italiane and 23 centimes.

Example for Illustration.

Reduce 14783 lire, 3 soldi, 9 denari, to pounds sterling, exchange at 59 lire piccole per pound sterling.

$$\begin{array}{c|l} 59 & 1 \\ \hline x & 14783{\cdot}1875 \end{array}$$

$$x = \frac{14783{\cdot}1875}{59} = £250{\cdot}5625 = £250 \quad 11 \quad 3. \text{ Ans.}$$

GENOA.

Accounts are kept here in lire, soldi, and denari, as in Venice.

Usance for bills drawn on Genoa from Amsterdam, Hamburgh, Spain, and Sicily, is two months, and from London and Lisbon three months, after date: from Naples, Ancona, and Trieste, twenty-two days' sight; Venice and Rome fifteen days; Augsburg and Verona fourteen days; Leghorn, Milan, and Turin, eight days' sight; Constantinople 30 days' sight.

TABLE.

12 denari = 1 soldo.
20 soldi = 1 lira, *fuori banco.*

The new coinage are gold pieces of 4 and 2 pistoles, of 96 and 48 lire, and the silver scudo of 8 lire, with halves and quarters.

NAPLES.

Accounts are kept here in ducati di regno, carlíni, and grani.

Usance from any part of the kingdom of Naples is fifteen days after acceptance; from Sicily, Genoa, Venice, Leghorn and Rome, ten days; from Spain two months; from London 3 months; three days of grace are allowed on all bills, except those drawn at sight.

TABLE.

10 grani	=	1 carlino.
10 carlini	=	1 ducato.

NOTE.—The public banks divide the Ducato into 5 Tari, of 20 Grani each.

Example for Illustration.

How much sterling money is equivalent to 1014 ducati 16 grani di regno, at 37½d. per ducato?

ducato 1	~~37·5~~
~~240~~	12·5
~~6~~	~~1014·16~~
~~3~~	~~25·354~~
x	12·677

$\therefore x = 12{\cdot}677 \times 12{\cdot}5 = 158{\cdot}4625 = £158\ 9\ 3$. Ans.

SICILY.

Accounts are kept in onzie, tari, and grani.

TABLE.

20 grani = 1 taro.
30 tari = 1 onzia.

The scudo is reckoned at 12 tari; the furino at 6 tari; and the ducat at 10 tari. The coins of Naples are also current here.

Usance. Bills drawn from England are at three months after date, and no days of grace.

Example for Illustration.

In 1377 onzie 14 grani, how much sterling money, at 10s. 7d. per onzie?

grani 3 ~~600~~	127d.
3 ~~240~~	
x	~~826214~~ grani.
	~~4131·07~~
	51·638375

$$\therefore x = \frac{51{\cdot}638375 \times 127}{9} = £728\ 13\ 6.\ \text{Ans.}$$

RUSSIA.

Accounts are kept at St. Petersburg in rubles, grieves, and copecks.

TABLE.

10 copecks = 1 grieve.
10 grieves = 1 ruble.

The ruble is also divided into $33\frac{1}{3}$ altins; and the copeck into 2 dunushkas or 4 polushkas.

Example for Illustration.

How much Russian money is equal to £131 1 1, at 9⅛d. per ruble ?

$$\begin{array}{r|l} \frac{73}{8}\text{d.} & \begin{array}{l}1 \text{ r.}\\ 8\end{array} \\ \hline x & 31453\text{d.} \end{array}$$

$$\therefore x = \frac{31453 \times 8}{73} = \overset{\text{r.}\ \ \text{cop.}}{3446{\cdot}90.} \text{ Ans.}$$

Example for Practice.

In £1349 15 sterling, how many rubles, rating the ruble at 49 stivers, and exchange at 33s. 9½d. Flemish per pound sterling ? Ans. 5584 rubles 92 copecks.

DENMARK.

Since 1813, accounts have been kept here in Regs-bank dollars, marks, and skillings.

Usance. There is no established usance ; there are eight days' grace.

TABLE.

12 skillings	=	1 mark.
8 marks	=	1 Regsbank dollar.

Example for Illustration.

Reduce 1176 dol. 6 m. 6 skil. to British money, exchange at 14 dol. 4 marks per pound sterling.

$$\begin{array}{r|l} \cancel{14{\cdot}5} & £1 \\ \cancel{2{\cdot}9} & \\ \hline x & \cancel{1176{\cdot}8125} \\ & 235{\cdot}3625 \end{array}$$

$35{\cdot}3625 \div 2{\cdot}9 = £81\ \ 3\ \ 2{\cdot}16$. Ans.

Example for Practice.

Reduce £81 3 2·16 to Danish money, exchange at 14 dol. 4 marks per pound sterling.

Ans. 1176 dol. 6 m. 6 skil.

SWEDEN.

Accounts are kept in Sweden in riksdalers, skillings, and fenings.

Usance is reckoned at one month after sight. Six days' grace are allowed for the payment of bills, in case of necessity.

TABLE.

12 fenings	=	1 skilling.
48 skillings	=	1 riksdaler.

Example for Illustration.

How much sterling money must be given for 9292 riksdalers 19 skillings, at 4 riksdalers 30 skillings per pound sterling?

sk.	
222	1
x	446035 sk.

$$x = \frac{446035}{222} = £2009\ 3\ 4.\ \text{Ans.}$$

Example for Practice.

Reduce £774 14 to Swedish money, at 5 riksdalers 40 skillings per pound sterling.

Ans. 4519 riks. 4 skil.

PRUSSIA, (BERLIN.)

Accounts are kept throughout the Electorate of Brandenburg, in rixdollars, good groschen, and pennings.

Usance for bills drawn on Berlin is fourteen days after acceptance, with three days' grace. Berlin draws on London and Paris at two months after date.

TABLE.

12 pennings = 1 groschen.
24 groschen = 1 rixdollar.

Example for Illustration.

How much Berlin money is equal to £115 13 4, exchange at $6\frac{3}{4}$ rixdollars per pound sterling?

$$\begin{array}{r|l} £1 & 27 \\ 4 & \\ \hline x & 115\cdot665 \end{array}$$

$115\cdot665 \times 27 = 3122\cdot955 \div 4 = 780$r-ds. $17\cdot73$ gr. Ans.

Example for Pratice.

Reduce 739 rixdollars, 17 gr. 3 pen. into pounds sterling, at 6 rixd. 16 gr. per pound sterling.

Ans. £110 19 $1\frac{3}{4}$.

KONINGSBURG.

Accounts are kept here in pennings, groschens, and guldens or florins.

Usance is 14 days after acceptance, including Sundays and holidays, with six days' grace.

TABLE.

18 pennings = 1 groschen.
30 groschen = 1 gulden.

A gulden is also reckoned at 8 good groschen, or 90s. Prussian money.

The Berlin coins are also current here.

VIENNA.

Accounts are kept here in guldens, creutzers, and pennings. Usance is fourteen days after acceptance, and days of grace are three.

TABLE.

4 pennings	=	1 creutzer.
60 creutzers	=	1 gulder or florin.
$1\frac{1}{2}$ florin	=	1 rixdollar of account.
2 florins	=	1 rixdollar specie.

Example for Illustration.

In 6451 florins 50 creutzers, how many pounds sterling; exchange at 9 florins 40 creutzers per pound sterling?

$$\begin{array}{c|c} 580 & 387110 \\ \hline x & £1 \end{array}$$

38711 ÷ 58 = £667 8 $7\frac{1}{2}$. Ans.

Example for Practice.

Reduce 9737 florins 30 creutzers into pounds sterling, at 10 florins per pound sterling. Ans. £973 15.

CONSTANTINOPLE.

Accounts are kept here in piastres or Turkish dollars, paras, and aspers.

Bills on Turkey are generally at 31 days' sight. They are usually paid on the day on which they become due.

TABLE.

3 aspers	=	1 para.
40 paras	=	1 piastre or dollar.*

Example for Illustration.

How much Turkish money is equal to £81 2 7, at 18 Turkish dollars per pound sterling.

$$\begin{array}{cc|cc} 40 & \cancel{240} & \cancel{18} & 3 \\ \hline & x & 19471 & \end{array}$$

$$\frac{19471 \times 3}{40} = 1460 \text{ dol. } 13 \text{ paras. Ans.}$$

Example for Practice.

Reduce 170 dollars 37 paras to pounds sterling, at 17 dollars 3 paras per pound sterling.

Ans. £10 0 2·4.

AMERICAN UNITED STATES.

Accounts are kept in the United States in dollars and cents.

The sterling value of the dollar is 4s. 6d.

TABLE OF THE MONEY OF THE UNITED STATES.

10 mills	=	1 cent.
10 cents	=	1 dime.
10 dimes	=	1 dollar.
10 dollars	=	1 eagle.†

* The piastre is called grouch by the Turks, and dollar by the English.

† The *eagle* contains 247¼ of pure, or 270 grains standard gold, the standard being $\frac{11}{12}$ fine. Its intrinsic value in English money

Example for Illustration.

Reduce 1000 dollars 75 cents, to sterling, exchange at par.

dollar 1	9 s.
~~2~~	
s. x	~~1000·75~~ dollars
~~20~~	~~200·15~~
~~4~~	~~100·075~~
	25·01875

$\therefore x = 25{\cdot}01875 \times 9 = £225{\cdot}16875$. Ans.

Examples for Practice.

1. A gentleman setting out for Amsterdam, wishes to convert into florins and cents £250, how many is he to receive, exchange being at 10 florins 84 cents per pound sterling. Ans. 2710 florins.

Example 2.

A traveller, on quitting Amsterdam, changes 7375 florins 27 cents into the currency of Vienna, how many florins and cruetzers is he to receive, exchange at 1 florin 75 cents per rixdollar of account.
Ans. 6321 flor. 39·6 creut.=4214 rixd. 0 flo. 39·6 cr.

Example 3.

The traveller changes 7626 rixd. 1 flor. 45 cruetzers into the currency of St. Petersburg, how many rubles and copecks is he to receive, exchange at 120 creutzers per ruble? Ans. 5720 rubles 37½ copecks.

is £2 3 8; half and quarter eagles are coined in the same proportion. The *dollar* contains 371¼ grains of pure silver; its intrinsic value is 4s. 3¾d. nearly.

Cent contains 208 grains of copper; half cents are in the same proportion.

Example 4.

From St. Petersburg the traveller sets out for Berlin, and changes 6145 rubles 45 copecks into the currency of Berlin, how many rixdollars, groschens, and pennings is he to receive, exchange at 1 ruble 75 copecks, per rixdollar? Ans. 3511 rixd. 16 gro. $5\frac{17}{35}$ pen.

Example 5.

The above traveller sets out from Berlin for Paris, and converts into francs and cents 6470 rixdollars 10 groschen, how much is he to receive, exchange at 19 rixdollars for 75 francs? Ans. 2541 fr. 12 cents.

Example 6.

If, for 492 rixdollars 10 groschen, Prussian money, I receive 1943 francs 75 cents, French money, what is the rate of exchange between the two countries?
Ans. 19 rixd. for 75 francs$=$3 fr. 94 cents 6 dimes, per rixdollar.

Example 7.

A merchant at Paris buys bills on Madrid for 7475 francs, how many dollars of exchange is he to receive, exchange being at 14 francs per 4 dollars of exchange?
Ans. 2135 dol. 5 reals $24\frac{2}{7}$ mar.

Example 8

Convert 140 dollars 3 reals 18 mar., Spanish money, into Lisbon money, exchange at 578 rees per dollar of exchange? Ans. 81 mil. 175 rees.

Example 9.

If, for 81 mil. 175 rees, Lisbon money, I get 140 dol. 3 reals 18 mar. Spanish money, what is the rate of exchange? Ans. 578 rees per dollar.

Example 10.

Reduce 2467 francs 38 cents, French money, into Neapolitan money, exchange at 4 fr. 25 cents per ducato de regno? Ans. 580 ducati 56 grani.

Example 11.

Convert £401 7 1 into American currency, at 4s. 6d. per dollar? Ans. 1783 dol. 78 cents.

Example 12.

Convert 1364 dol. 60 cents, American money, into francs and cents, French money, exchange 5 fr. 30 cents per dollar. Ans. 7232 fr. 38 cents.

Example 13.

Convert 1756 francs, French money, into Turkish money, exchange at 1 fr. 38 cents per piastre?
Ans. 1272 pias. 18 par. 2 aspers.

Example 14.

How much French money is equivalent to 6451 Turkish dollars 20 paras, exchange at 1 fr. 38 cents per dollar? Ans. 8903 fr. 7 cents.

Example 15.

Convert 3737 florins 48 cruetzers, Austrian money, into Turkish money, exchange at 50 paras per florin? Ans 4672 dol. 10 paras.

Example 16.

In 1272 pias. 8 par. 2 aspers, Turkish Money, how many francs and cents, French money, exchange 1 fr. 38 cents per piastre? Ans. 1756 francs.

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